BEIHANG UNIVERSITY

LITERATURE SHARING

Accommodating unobservability to control flight attitude with optic flow

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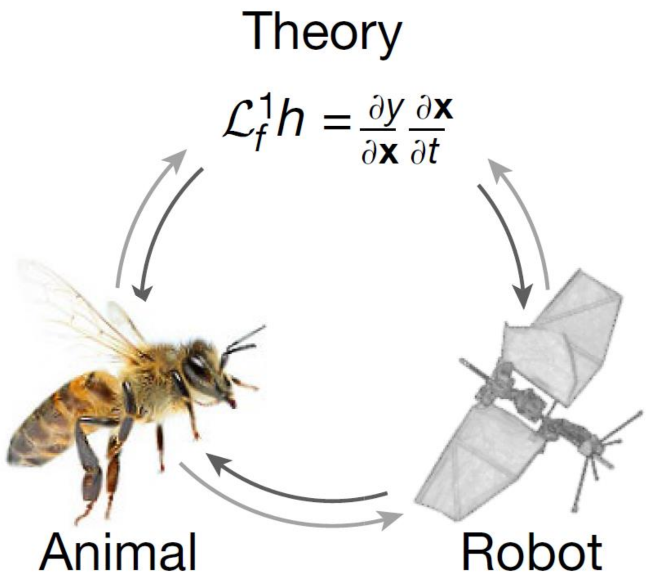
CONTENT



Theory
$\mathcal{L}_f^1 h = \frac{\partial y}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial t}$
Animal
Robot

PROBLEM

Attitude Control

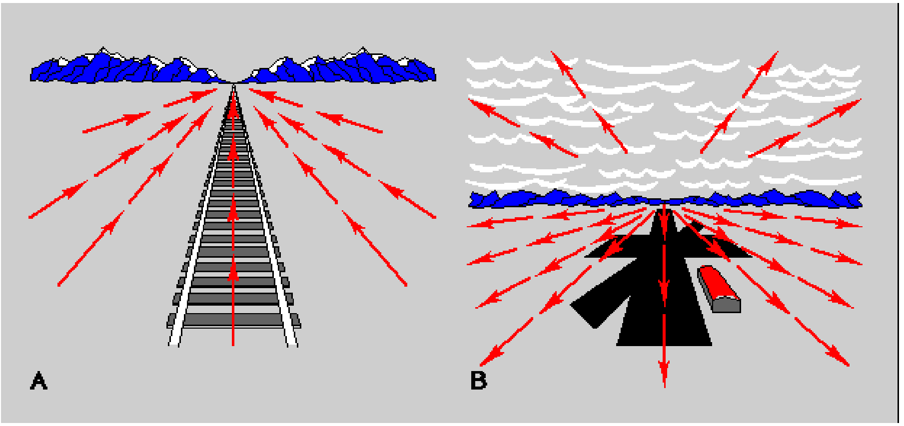
- Accelerator -> gravity direction
- Gyro -> body rate

- ~~Accelerator -> gravity direction~~
- Gyro -> body rate
- Optic flow

What is optic flow?

- the distribution of apparent velocities of movement of brightness pattern in an image

--- Wikipedia



Can we extract the attitude information from optic flow?

CONSTANT-HEIGHT MODEL

State $\vec{x} = [v_I, \varphi, Z_I]$ **Control Input** $u = p$

System model

$$f(\vec{x}, u) = \begin{bmatrix} \dot{v}_I \\ \dot{\varphi} \\ \dot{Z}_I \end{bmatrix} = \begin{bmatrix} g\,tan(\varphi) \\ p \\ 0 \end{bmatrix}$$

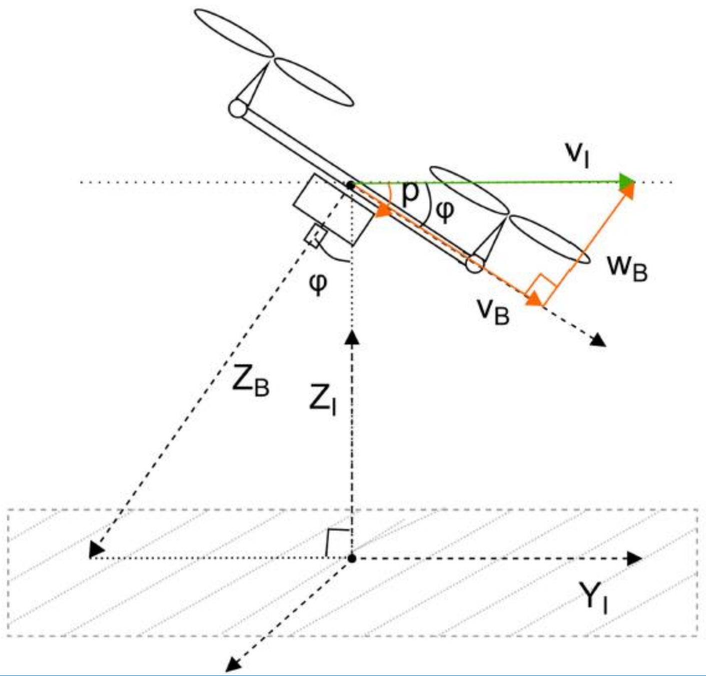
Measurement model

$$\omega_y = -\frac{v_B}{Z_B} + p = -\frac{cos^2(\varphi) v_I}{Z_I} + p$$

$$\Rightarrow \quad y = \omega_y = h(\vec{x})$$

Assumption

- Consider Y-Z plane only
- Altitude is constant



Observability Analysis

Observability analysis

$$\dot{y} = \frac{\partial y}{\partial t} = \mathcal{L}_f^1 h = \frac{\partial y}{\partial \vec{x}}\frac{\partial \vec{x}}{\partial t} = \frac{(2pv_I - g)sin(2\varphi)}{2\,Z_I}$$

$$\ddot{y} = \mathcal{L}_f^2 h = \frac{\partial \mathcal{L}_f^1 h}{\partial \vec{x}}\frac{\partial \vec{x}}{\partial t} = p\frac{2\,p\,v_I\,cos(2\varphi)\ +\ g\ -\ 2\,g\,cos(2\varphi)}{Z_I}$$

“observability mapping”

$$\mathrm{H}(\vec{x}) = \begin{bmatrix} h \\ \mathcal{L}_f^1 h \\ \mathcal{L}_f^2 h \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -\frac{cos^2(\varphi)v_I}{Z_I} + p \\ \frac{(2pv_I - g)sin(2\varphi)}{2\,Z_I} \\ p\frac{2(pv_I - g)cos(2\varphi) + g}{Z_I} \end{bmatrix}$$

observability matrix

$$\mathcal{O} = \frac{\partial \mathrm{H}(\vec{x})}{\partial \vec{x}} = \begin{bmatrix} \frac{\partial y}{\partial \vec{x}} & \frac{\partial \dot{y}}{\partial \vec{x}} & \frac{\partial \ddot{y}}{\partial \vec{x}} \end{bmatrix} = \begin{bmatrix} -\frac{cos^2(\varphi)}{Z_I} & \frac{p\ sin(2\varphi)}{Z_I} & \frac{2\,p^2\,cos(2\varphi)}{Z_I} \\ \frac{sin(2\varphi)v_I}{Z_I} & \frac{(2pv_I - g)cos(2\varphi)}{Z_I} & 4p\frac{(g - pv_I)sin(2\varphi)}{Z_I} \\ \frac{cos^2(\varphi)v_I}{{Z_I}^2} & \frac{(g - 2pv_I)sin(2\varphi)}{2{Z_I}^2} & -p\frac{2(pv_I - g)cos(2\varphi) + g}{{Z_I}^2} \end{bmatrix}$$

$$\mathcal{O}^T\,\mathrm{d}\vec{x} = \begin{bmatrix} dy \\ d\dot{y} \\ d\ddot{y} \end{bmatrix} \qquad \mathrm{d}\vec{x} = \mathcal{O}^{-T}\begin{bmatrix} dy \\ d\dot{y} \\ d\ddot{y} \end{bmatrix}$$

Full rank → **locally, weakly observable**

Kou, Shauying R., David L. Elliott, and Tzyh Jong Tarn. "Observability of nonlinear systems." *Information and Control* 22.1 (1973): 89-99.

OBSERVABILITY ANALYSIS

Special cases

$p = 0$ ✗

$$\mathcal{O}' = \begin{bmatrix} -\frac{\cos^2(\varphi)}{Z_I} & 0 & 0 \\ \frac{\sin(2\varphi)v_I}{Z_I} & \frac{-g\cos(2\varphi)}{Z_I} & 0 \\ \frac{\cos^2(\varphi)v_I}{Z_I^2} & \frac{g\sin(2\varphi)}{2Z_I^2} & 0 \end{bmatrix}$$

$v_I = 0$ ✓

$$\mathcal{O}' = \begin{bmatrix} -\frac{\cos^2(\varphi)}{Z_I} & \frac{p\sin(2\varphi)}{Z_I} & \frac{2p^2\cos(2\varphi)}{Z_I} \\ 0 & \frac{-g\cos(2\varphi)}{Z_I} & 4p\frac{g\sin(2\varphi)}{Z_I} \\ 0 & \frac{g\sin(2\varphi)}{2Z_I^2} & -p\frac{-2g\cos(2\varphi)+g}{Z_I^2} \end{bmatrix},$$

$\varphi = 0$ ✓

$$\mathcal{O}'' = \begin{bmatrix} -\frac{1}{Z_I} & 0 & \frac{2p^2}{Z_I} \\ 0 & \frac{-g}{Z_I} & 0 \\ 0 & 0 & \frac{pg}{Z_I^2} \end{bmatrix},$$

- Any condition when p=0 → unobservable
- A perfect hover → rate=0 → unobservable

All conditions

$$|\mathcal{O}| = -\frac{g\,p\left(\frac{\cos(2\phi)}{2}+\frac{1}{2}\right)(g\cos(2\phi) - 2g + 2pv\cos(2\phi))}{Z_I^4} = 0$$

$p = 0$ $\quad \lim_{Z_I\to\infty} |\mathcal{O}| = 0$ $\quad g = 0$

MATLAB symbolic toolbox

$$p = \frac{2g - g\cos(2\phi)}{2v\cos(2\phi)} \qquad \varphi = \frac{1}{2}\pi \qquad \varphi = \frac{\operatorname{acos}\left(\frac{2g}{g+2pv}\right)}{2}$$

$$g^2 \le 4p^2v^2 \wedge \frac{4p^2v^2}{3} \le 3\left(g+\frac{4pv}{3}\right)^2 \qquad \varphi = -\frac{\operatorname{acos}\left(\frac{2g}{g+2pv}\right)}{2} \qquad v_I = \frac{2g - g\cos(2\phi)}{2p\cos(2\phi)}$$

Quite unlikely to occur!

Numerical verification

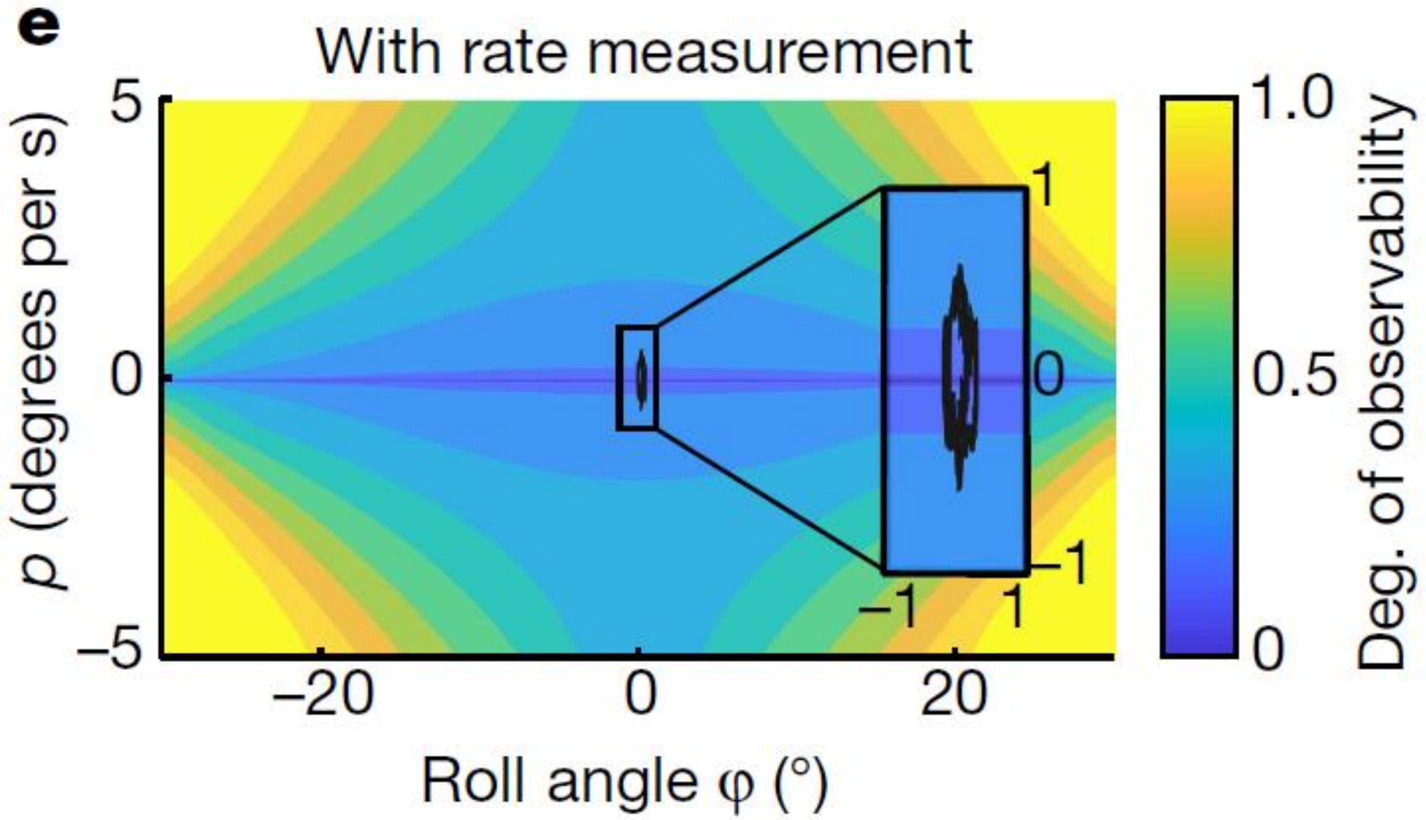
Degree of observability

$$\kappa(\mathcal{O}) = \frac{s_{max}(\mathcal{O})}{s_{min}(\mathcal{O})}$$

S → Singular value

$$d(\mathcal{O}) = \frac{1}{log(\kappa(\mathcal{O}))}$$

- 0: Lower observability
- 1: Higher observability



Attitude is observable! However, not when hovering still.

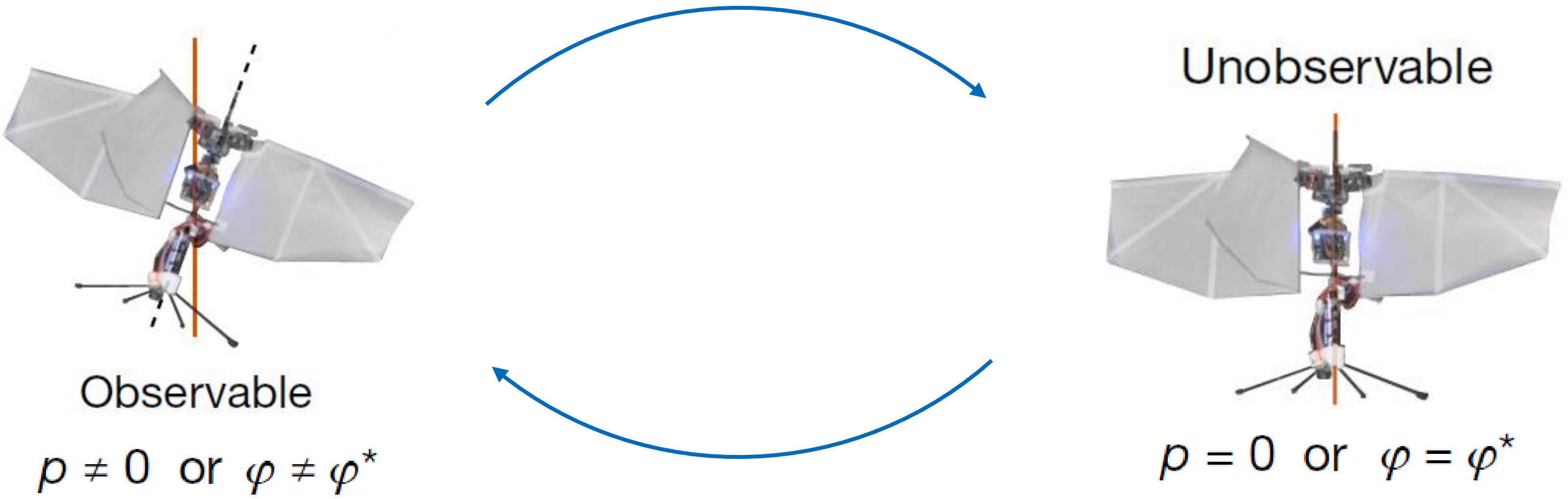
Stability Analysis

Attitude is observable! However, not when hovering still.

What if you want to hover?

This leads to unobservability!

Is it possible to control the drones to hover?



PROOF OF STABILITY

$$\vec{x} = [v_I, \varphi, Z_I]$$

$$u = p$$

$$f(\vec{x}, u) = \begin{bmatrix} \dot{v}_I \\ \dot{\varphi} \\ \dot{Z}_I \end{bmatrix} = \begin{bmatrix} g\ tan(\varphi) \\ p \\ 0 \end{bmatrix}$$

Part I: Stable control will lead the observable system to the desired attitude, with zero rate

Lyapunov function $V = (\varphi - \varphi^*)^2$

$$\dot{V} = \frac{\partial V}{\partial t} = \frac{\partial V}{\partial \vec{x}} \frac{\partial \vec{x}}{\partial t} = [0 \quad 2(\varphi - \varphi^*) \quad 0] \begin{bmatrix} g\ tan(\varphi) \\ p \\ 0 \end{bmatrix} = 2p(\varphi - \varphi^*)$$

$$p = -K(\varphi - \varphi^*), K > 0, \Rightarrow \frac{\partial V}{\partial t} = -2K(\varphi - \varphi^*)^2 < 0$$

Part II: Unobservable conditions always lead to observable conditions

- Measurement noise
- Actuation noise
- External disturbance

Remark 1

Q: Asymptotic stability for a delay-less control system

A: Attitude control with basic PID is applied widely and successfully

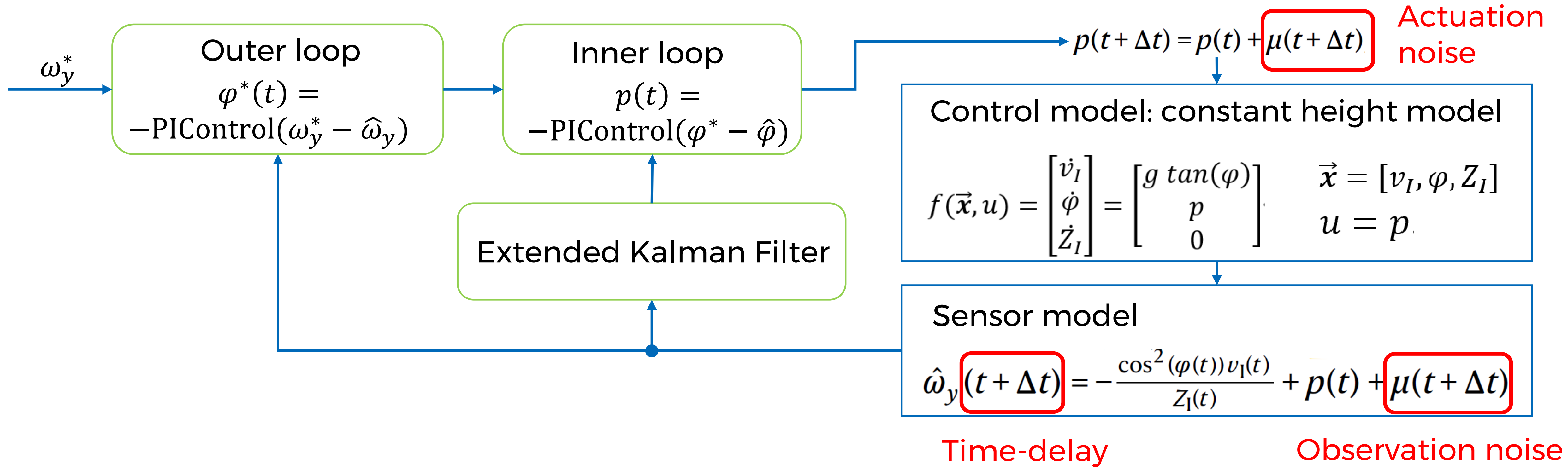
$p = 0$ or $\varphi = \varphi^*$

Remark 2

Q: The effect of outer loop controller is not considered.

A: **Part I** → nested Lyapunov analysis; **Part II** → more noise, more possibility to induce observability

SIMULATION SETUP



1. No noise, no disturbances, no observability, no problem

2. Observation noise
3. Actuation noise
4. Lateral disturbances
5. Malicious disturbances

- *Control target: hover, i.e.* $\omega_y^* = 0$

1. No noise, no disturbances, no observability, no problem

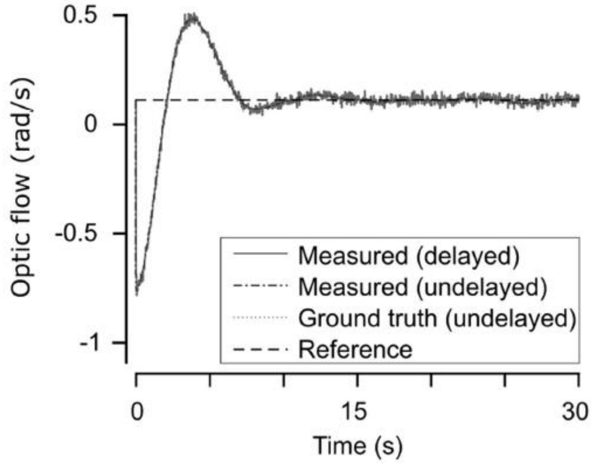
2. Observation noise

3. Actuation noise

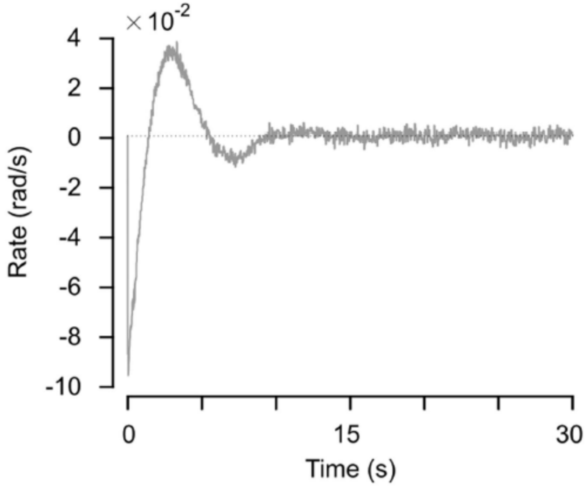
4. Lateral disturbances

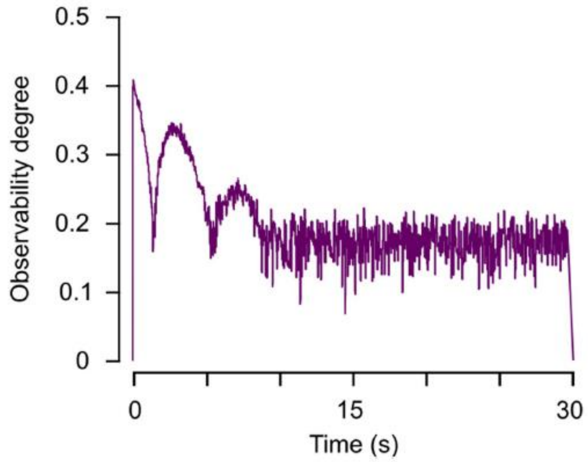
5. Malicious disturbances

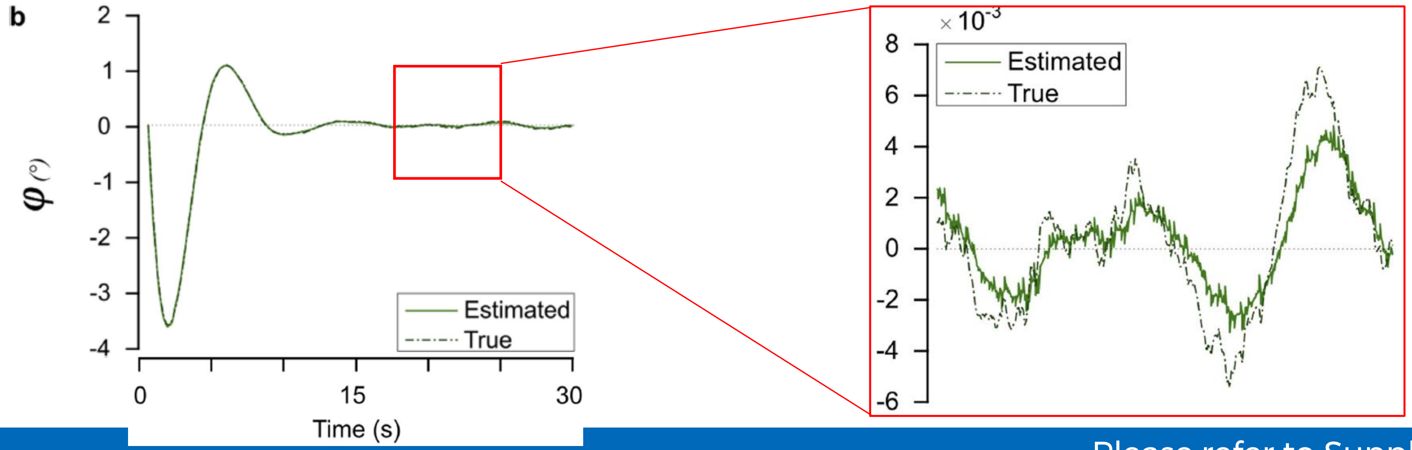
$\widehat{\omega}_y$

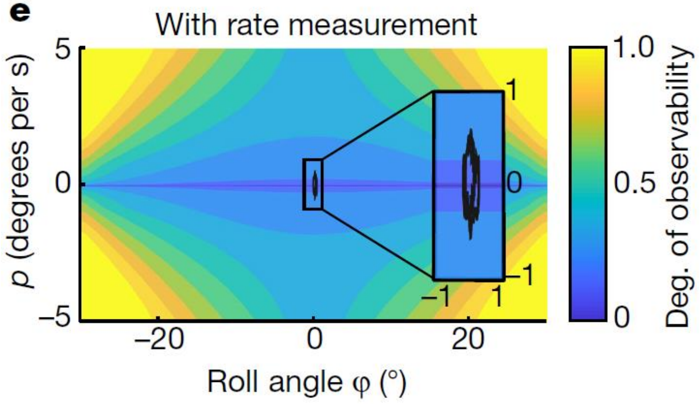


p









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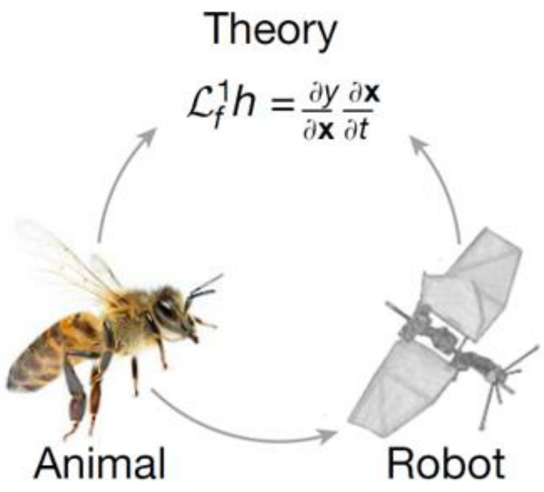

Theory
$\mathcal{L}_f^1 h = \frac{\partial y}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial t}$
Animal
Robot

Model Extensions

Constant height model

$$f(\vec{x},u)=\begin{bmatrix}\dot{v}_I\\ \dot{\varphi}\\ \dot{Z}_I\end{bmatrix}=\begin{bmatrix}g\tan(\varphi)\\ p\\ 0\end{bmatrix}\qquad \vec{x}=[v_I,\varphi,Z_I]\qquad u=p$$

Constant height model without rate measurements

$$\begin{bmatrix}\dot{v}_I\\ \dot{\varphi}\\ \dot{p}\\ \dot{Z}_I\end{bmatrix}=\begin{bmatrix}g\tan(\varphi)\\ p\\ M/I\\ 0\end{bmatrix}\qquad \vec{x}=[v_I,\varphi,p,Z_I]\qquad u=M$$

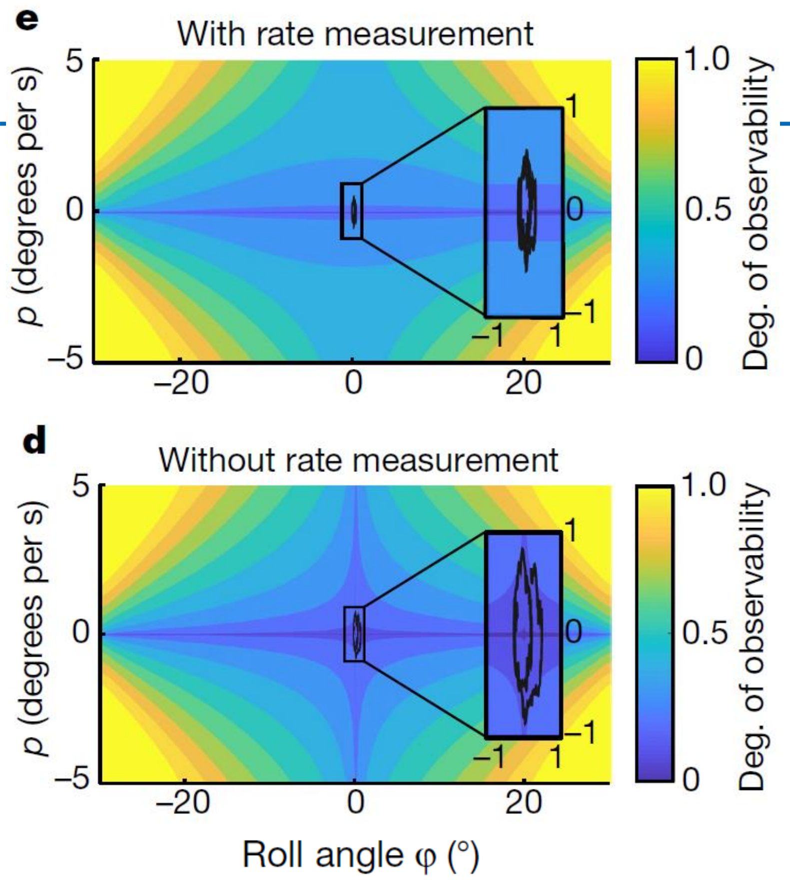
Varying height model with drag and wind

Varying height model with thrust bias and optic flow divergence

Surface with a slope

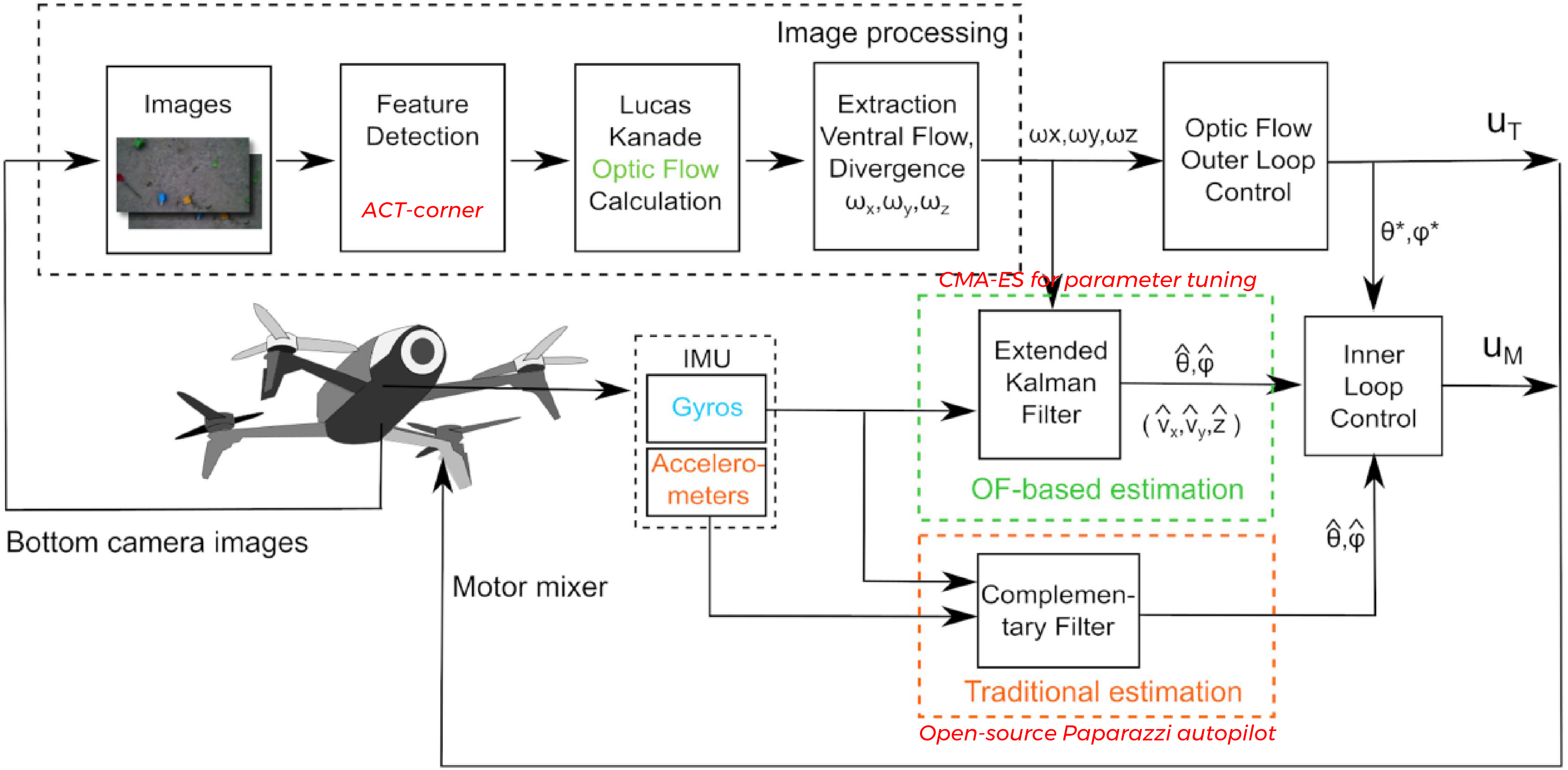
Optic-flow-based attitude estimation in generic environments

Model with independently moving head and body



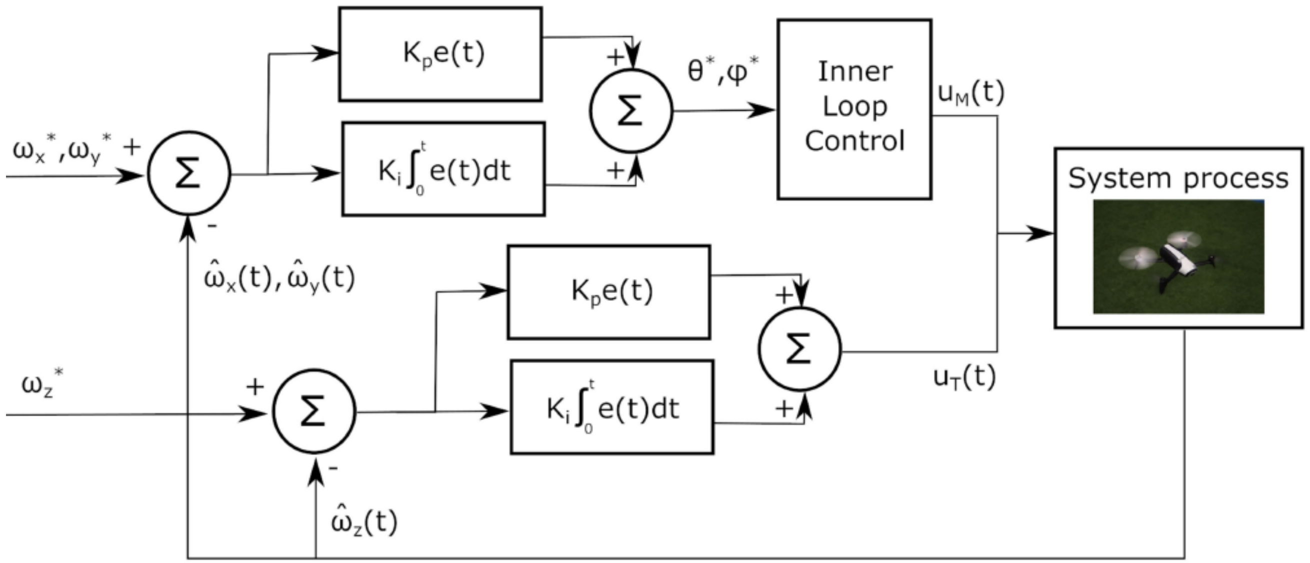
Locally, weakly observable, except for the hover condition

Onboard the drone

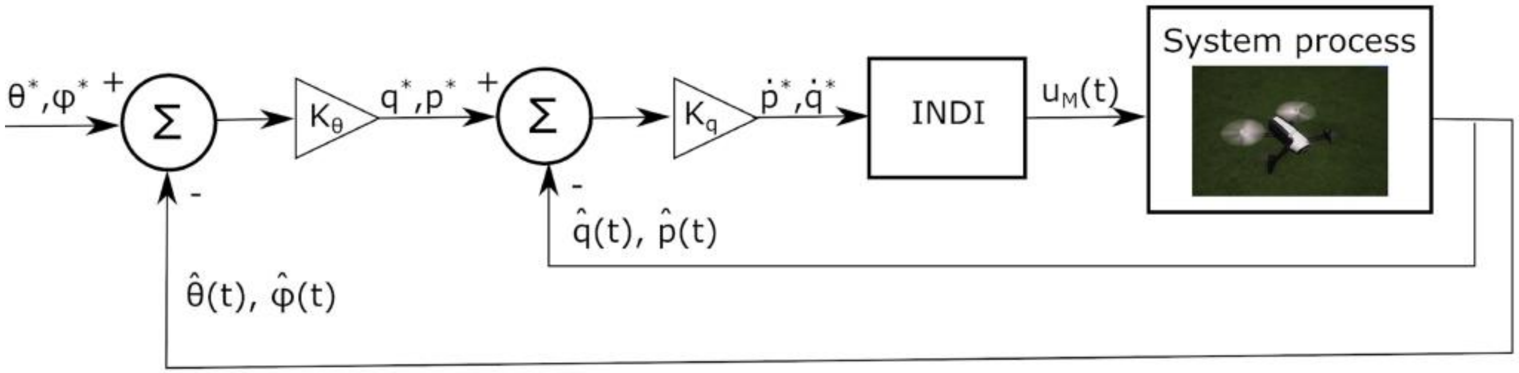


QUAD ROTOR EXPERIMENT

Optic flow outer loop control



Inner loop control



Quad Rotor Experiment

Supplementary video 1

Quadrotor flying with optic-flow-based attitude

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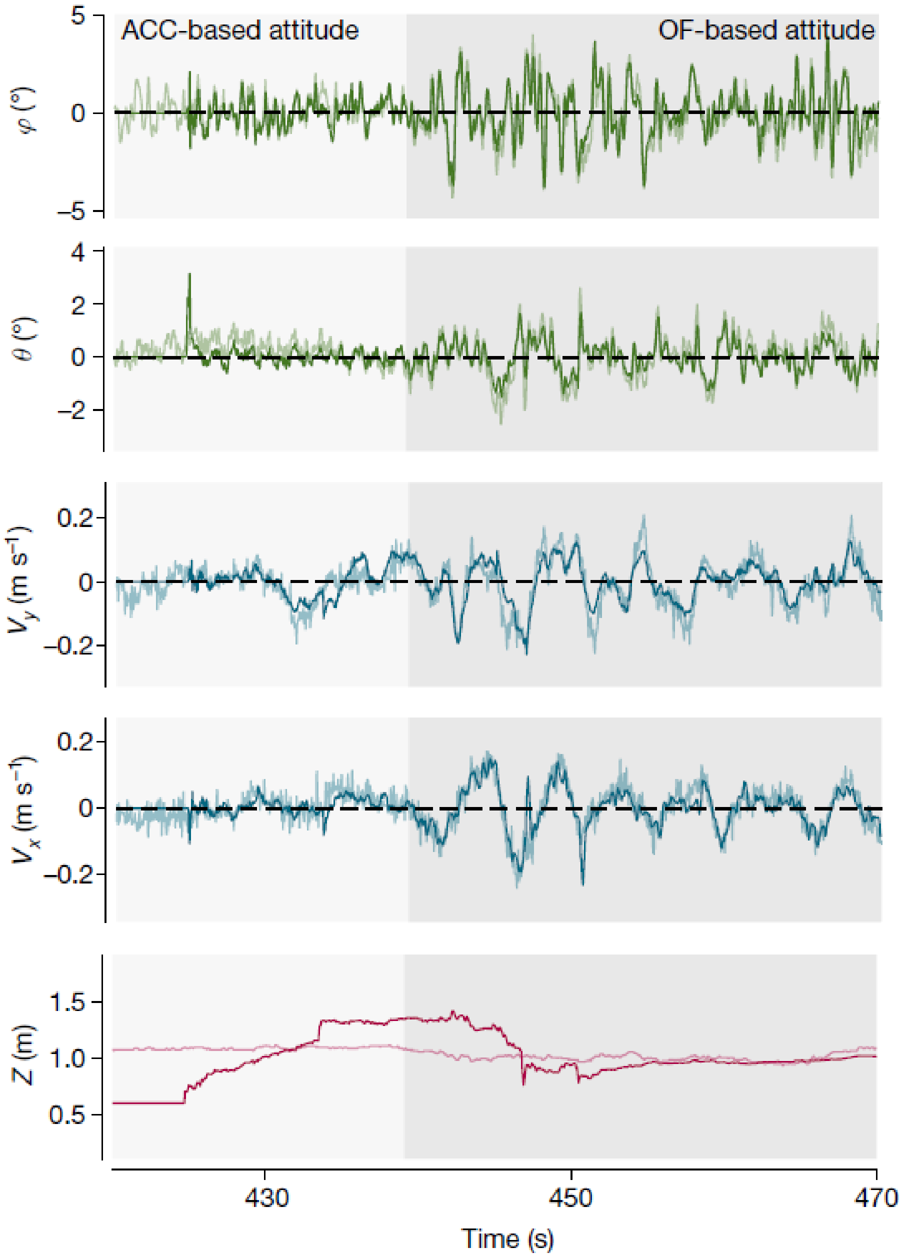
QUAD ROTOR EXPERIMENT

Constant model | **Varying height model**

Result

Flat ground

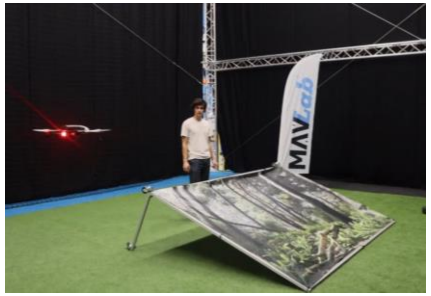


Slope surface

3D structure

Disturbance

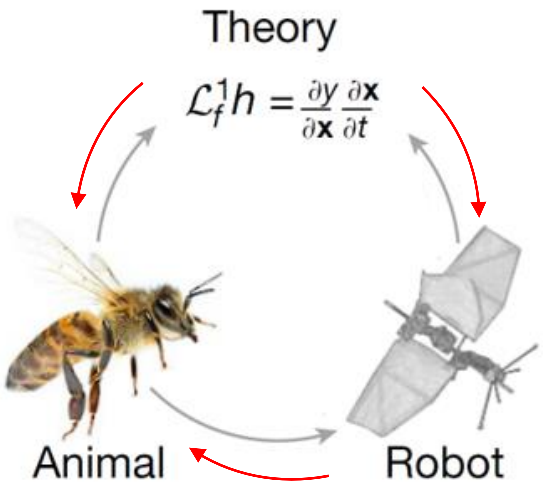
A tilted slope



A moving tilted slope

3D structure

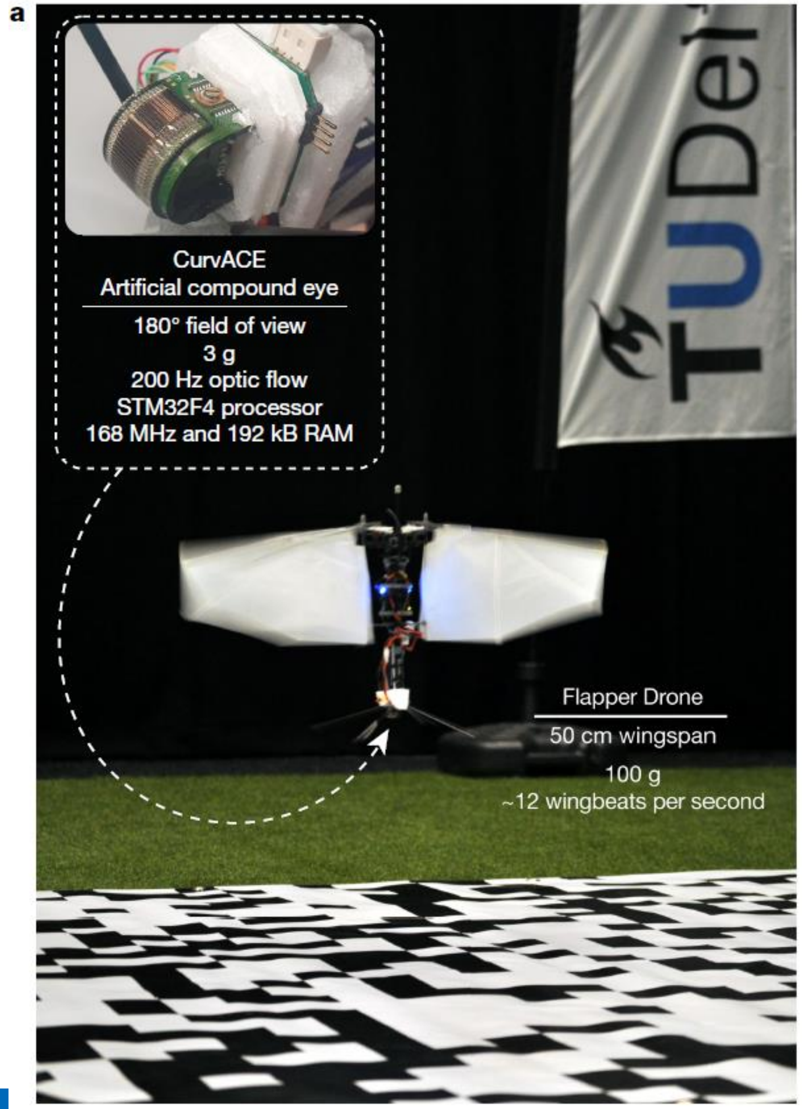
Content

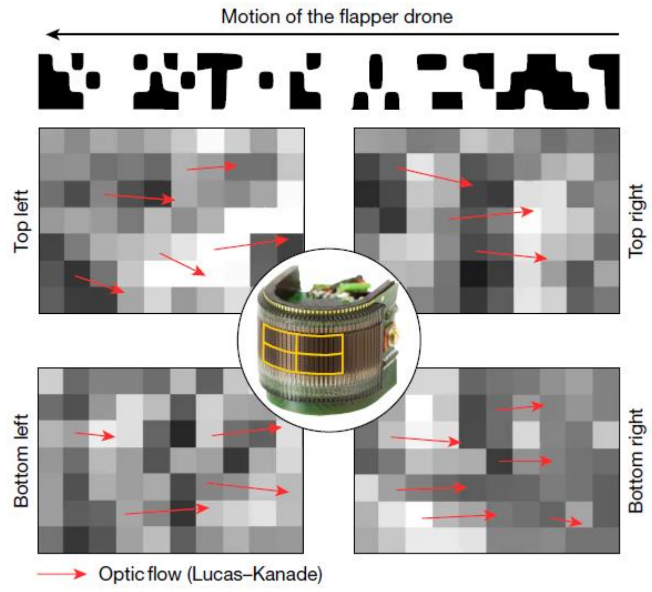


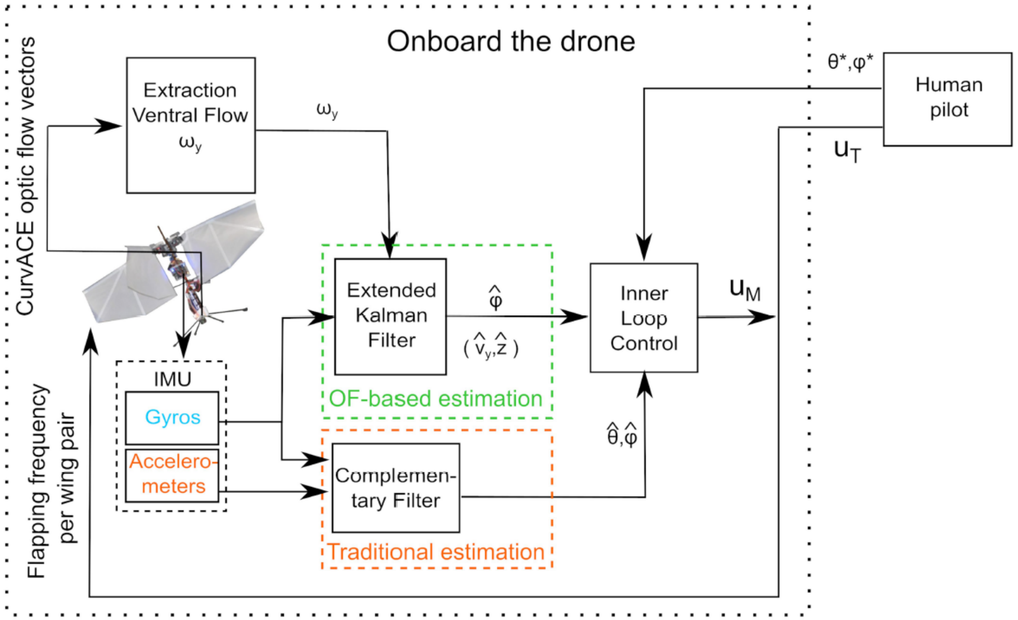
What can drones teach us about nature?

Flapping-Wing Robot Experiment

What can drones teach us about nature?







Supplementary video 2

Flapper drone flying with optic-flow-based attitude

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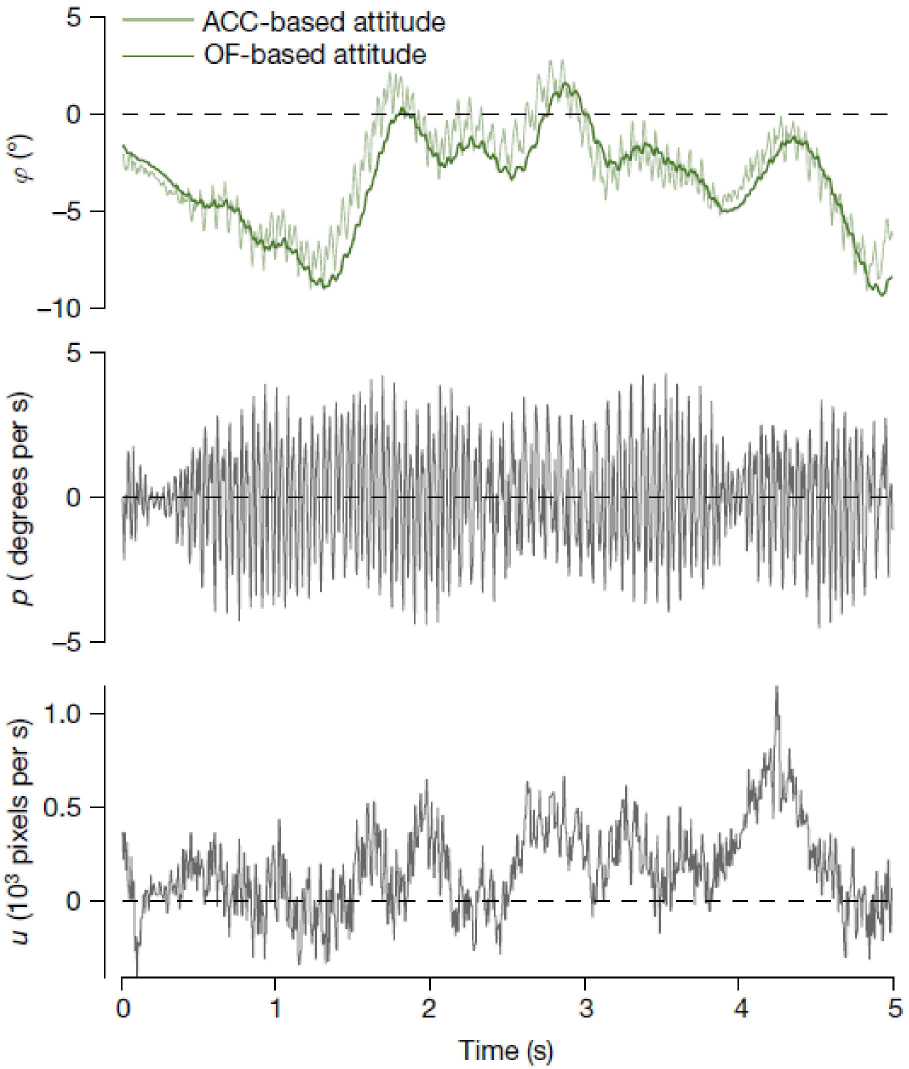
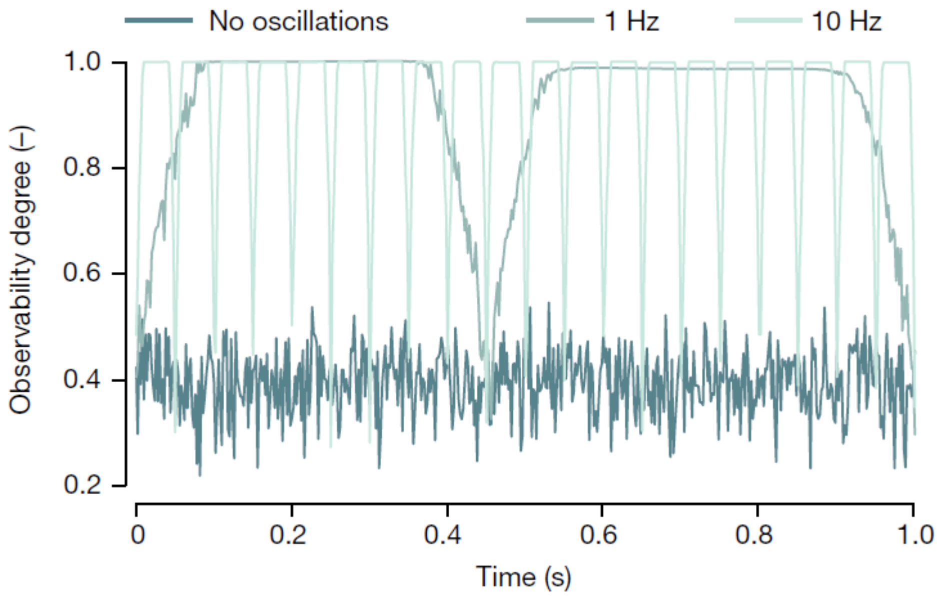
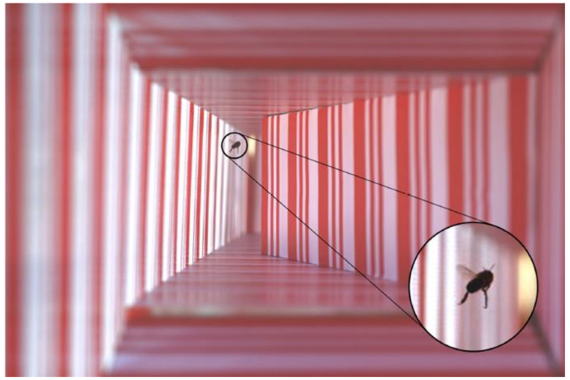
FLAPPING-WING ROBOT EXPERIMENT

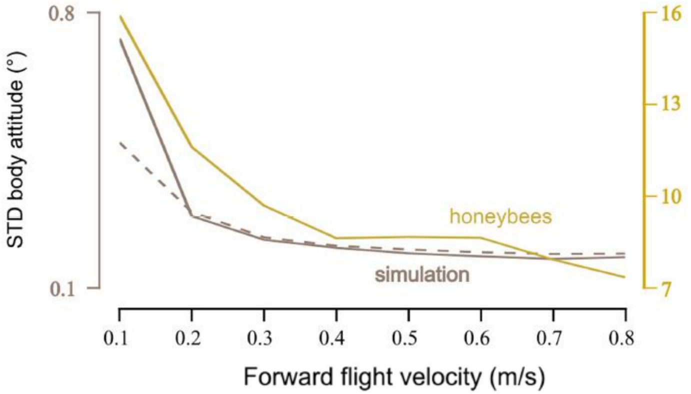
Result





The residual flapping motion improve attitude observability!





The results are close to those with honeybees

CONCLUSION

Can we extract the attitude information from optic flow?

Yes !

1. PROBLEM
2. CONSTANT-HEIGHT MODEL & 6 OTHER MODELS
3. OBSERVABILITY & SIMULATION
4. STABILITY & SIMULATION
5. QUAD ROTOR EXPERIMENT
6. FLAPPING-WING ROBOT EXPERIMENT

What can we learn from this article?

1. Standard process for control research

    Observability, Stability, Symbolic calculation, Simulation with delay and noise, Hardware experiment

2. A perfect example on how to go **deeper**
    - from a very simple model → a much complicated one
    - from special cases → general conditions
    - from simulation → real-world flight
    - from quadrotor → flappy robot

3. Where do they find the problem

4. Based on many basic algorithms: ACT-corner, Lucas Kanade, INDI, EKF, CMA-ES

- Data available: https://doi.org/10.4121/20183399
- Code available: https://github.com/tudelft/paparazzi/releases/tag/v5.17.5_attitude_flow
- Talk by Prof. Guido: https://collegerama.tudelft.nl/Mediasite/Play/12ead0e273964c1e9e63ca9d04bbb1a61d

Thanks