



# Nonlinear MPC for Quadrotors in Close-Proximity Flight with Neural Network Downwash Prediction

**Jinjie LI**, Liang HAN\*, Haoyang YU, Yuheng LIN, Qingdong LI, Zhang REN

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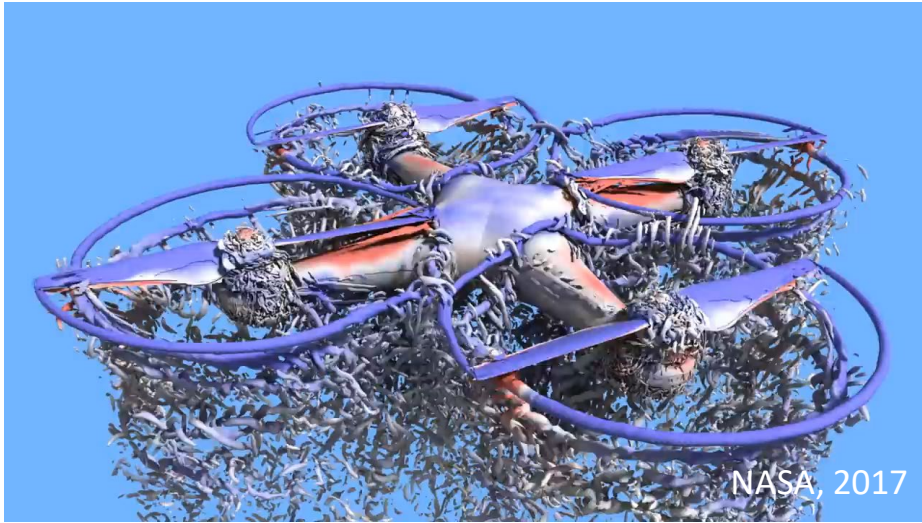
Open Source!  
Scan here!



[https://github.com/Li-Jinjie/ndp\\_nmmpc\\_qd](https://github.com/Li-Jinjie/ndp_nmmpc_qd)

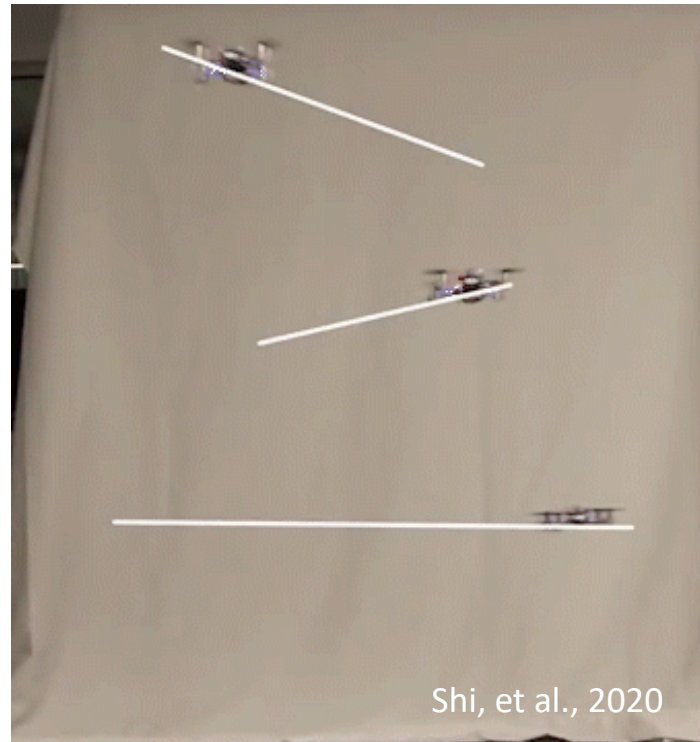
- 1 Introduction**
- 2 System overview**
- 3 Methodology**
- 4 Experiments**
- 5 Conclusion**

## Downwash effect?



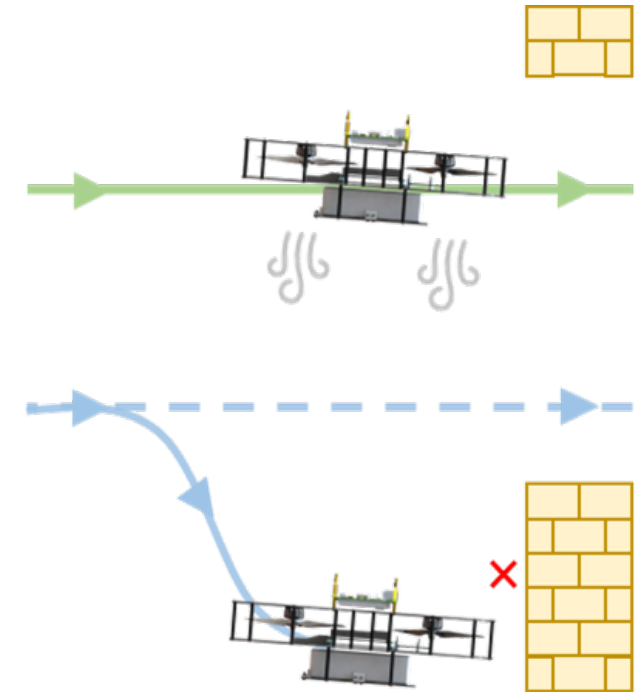
NASA, 2017

- CFD simulation for a DJI Phantom 3 quadrotor



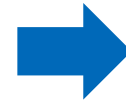
Shi, et al., 2020

- Downwash in Swarm Robotics

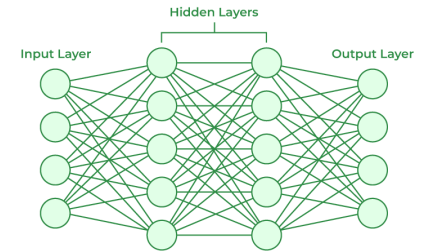


## Downwash effect

- Caused by other quadrotors
- Difficult to describe



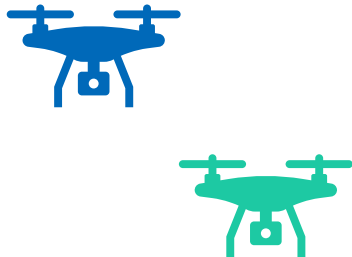
Neural Network



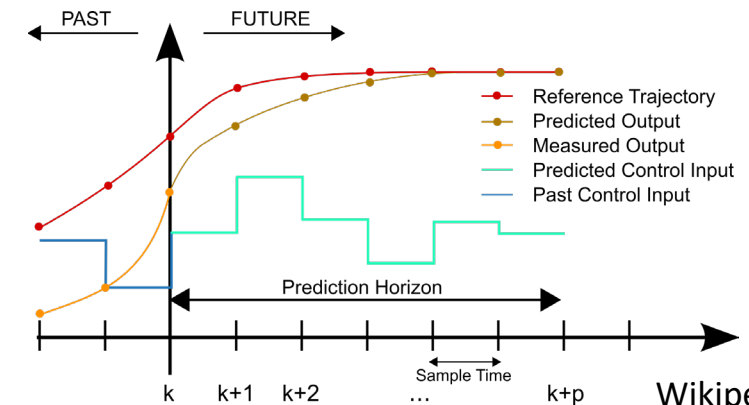
Model Predictive Control

## Control Problem

1. No prediction
2. No constraints

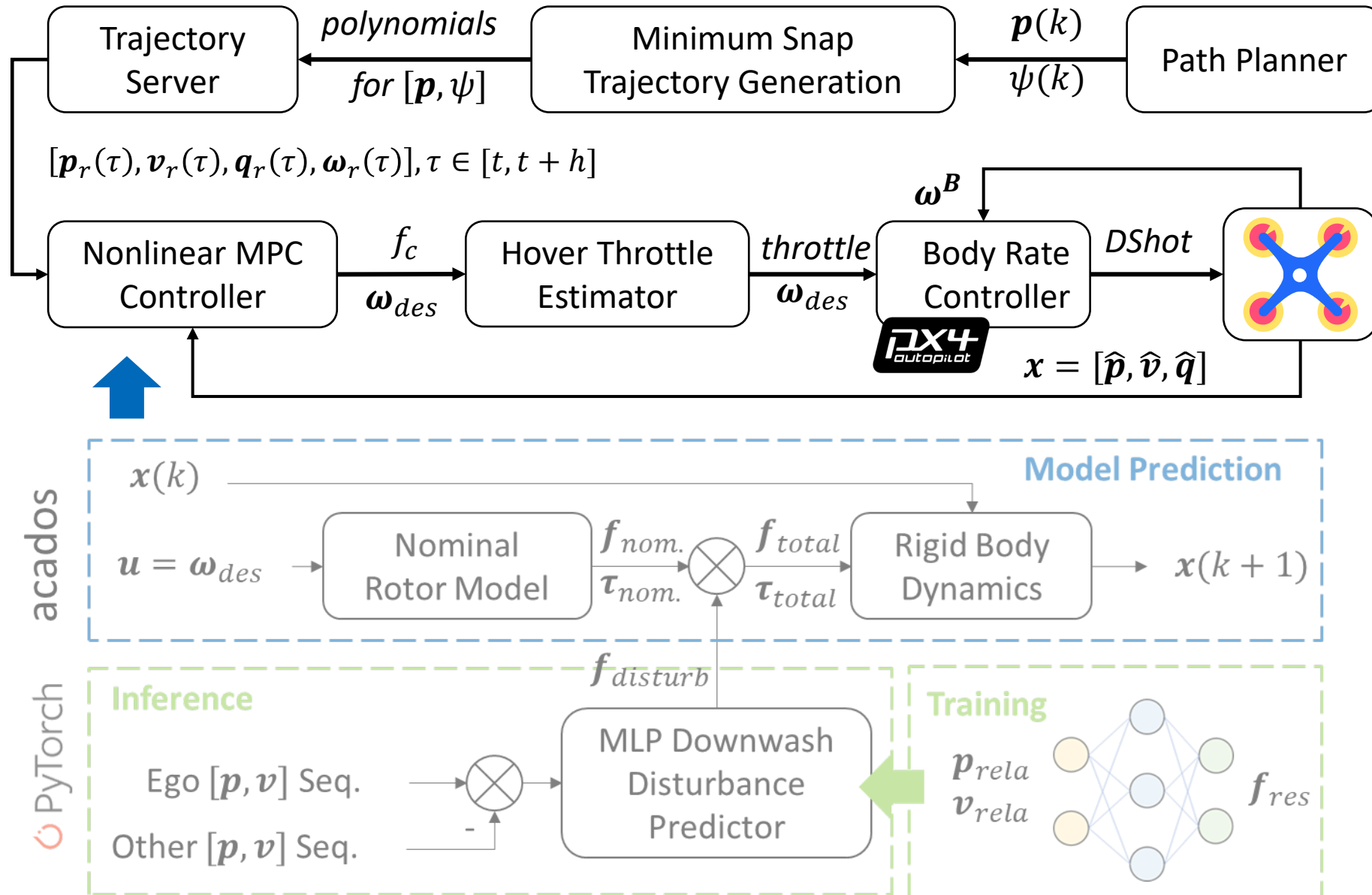


VS

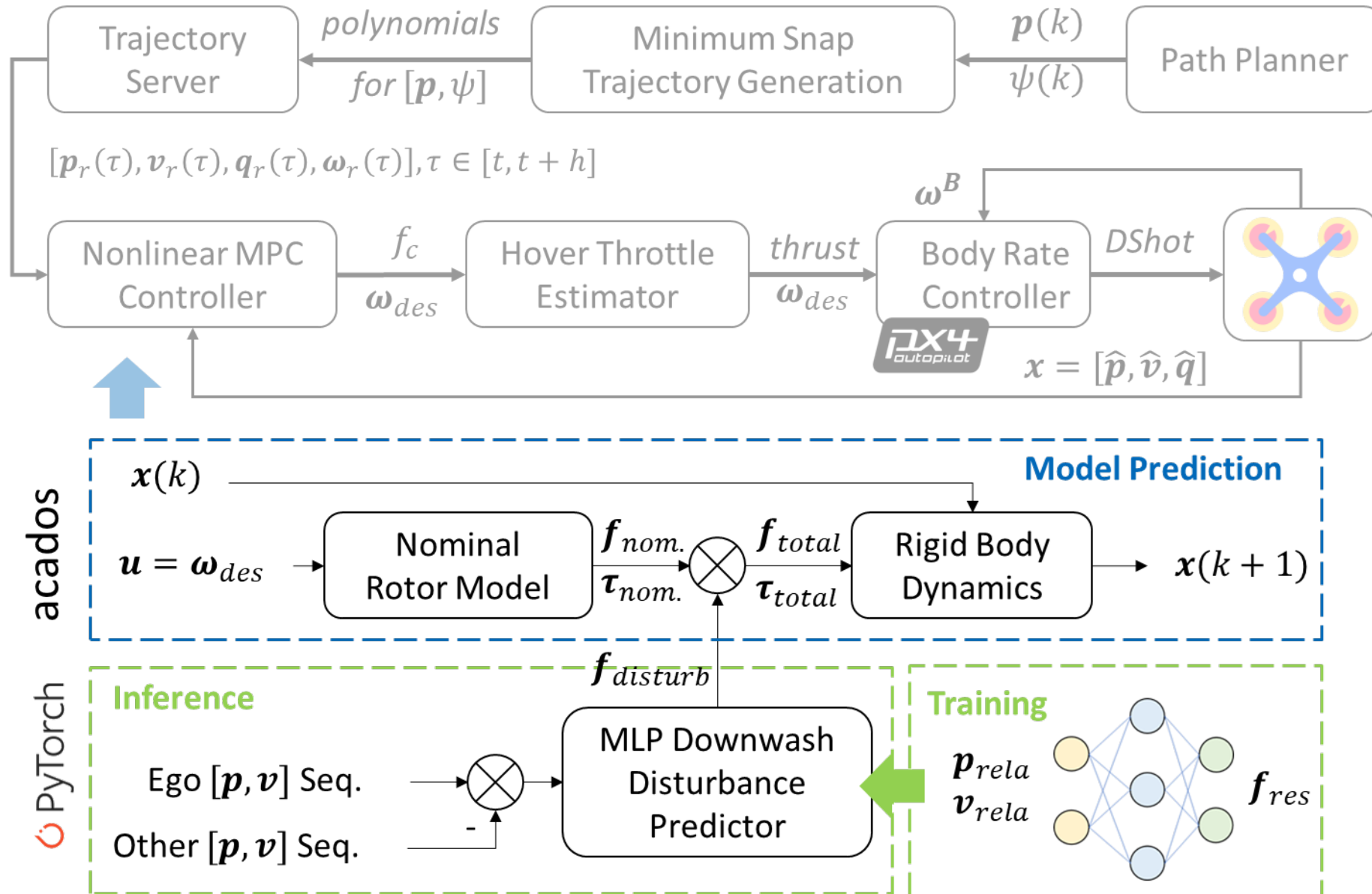


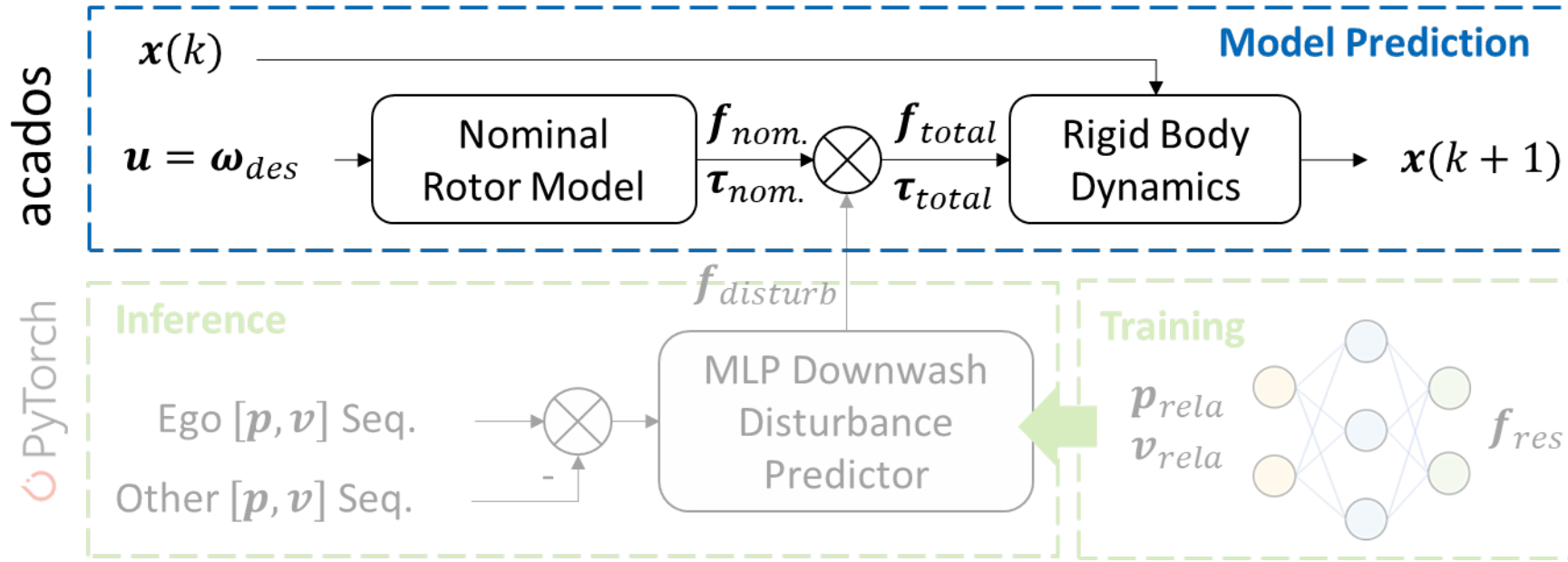
Wikipedia, 2023

# System Overview



# System Overview





State:  $p, v, q$

Input:  $\omega$  + Collective Thrust

## Nonlinear Model

$$\begin{cases} {}^I \dot{p} = {}^I v \\ {}^I \dot{v} = {}^I_B R(q) \cdot {}^B f_u / m + {}^I g \\ {}^B_I \dot{q} = 1/2 \cdot {}^B_I q \circ \mathcal{V}^* ({}^B \omega) \end{cases}$$

## Nonlinear Least Square Cost

$$\min_{u_k} \left( \bar{x}_N^T Q_N \bar{x}_N + \sum_{k=0}^{N-1} (\bar{x}_k^T Q \bar{x}_k + \bar{u}_k^T R \bar{u}_k) \right)$$

## Linear Constraints

$$x_0 = x_{init},$$

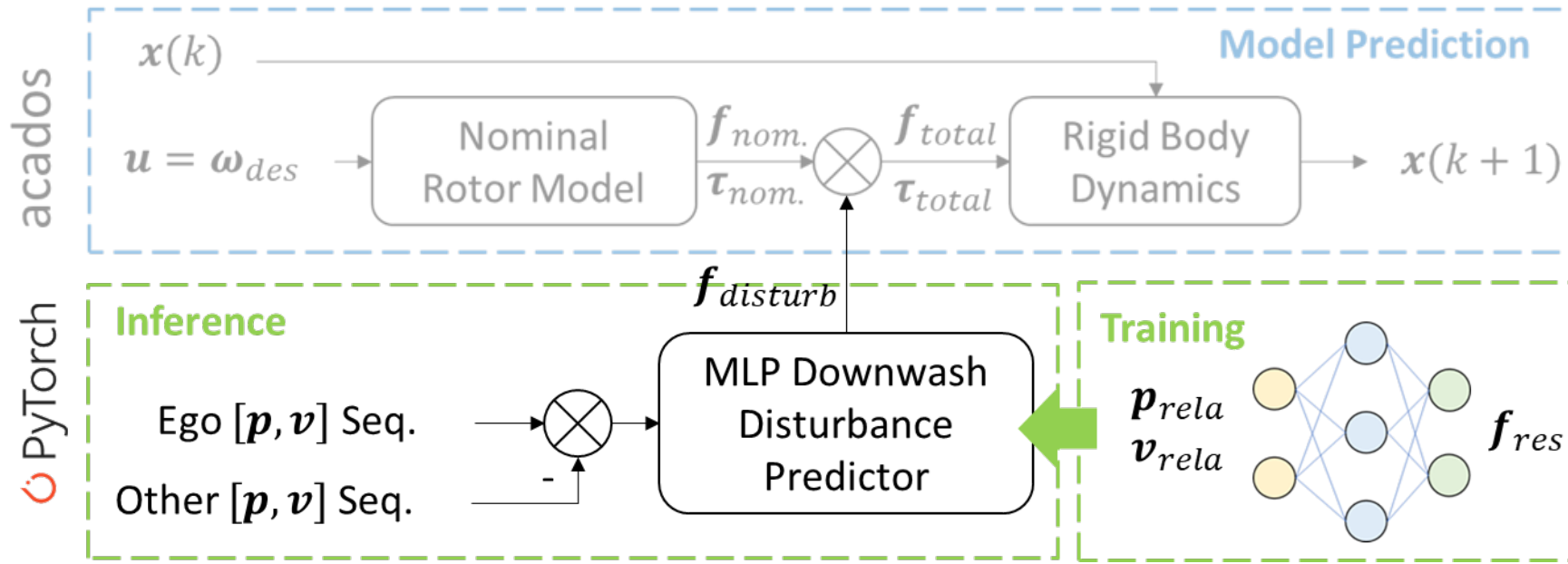
$$u_k \in [u_{min}, u_{max}],$$

$$q_e = q \circ q_r^{-1}$$

$$\bar{q}_k^T Q_q \bar{q}_k = \|\text{sgn}(q_{ew}) \cdot \mathcal{V}(q_e)\|_Q^2 = \mathcal{V}(q_e)^T Q_q \mathcal{V}(q_e),$$

Control Frequency:

50~60Hz



State:  $p, v, q$

Input:  $\omega$  + Collective Thrust

## Nonlinear Model

$$\begin{cases} {}^I \dot{p} = {}^I v \\ {}^I \dot{v} = {}^I_B R(q) \cdot {}^B f_u / m + {}^I g \\ {}^B_I \dot{q} = 1/2 \cdot {}^B_I q \circ \mathcal{V}^* ({}^B \omega) \end{cases}$$

## Nonlinear Least Square Cost

### Linear Constraints

$$x_0 = x_{init},$$

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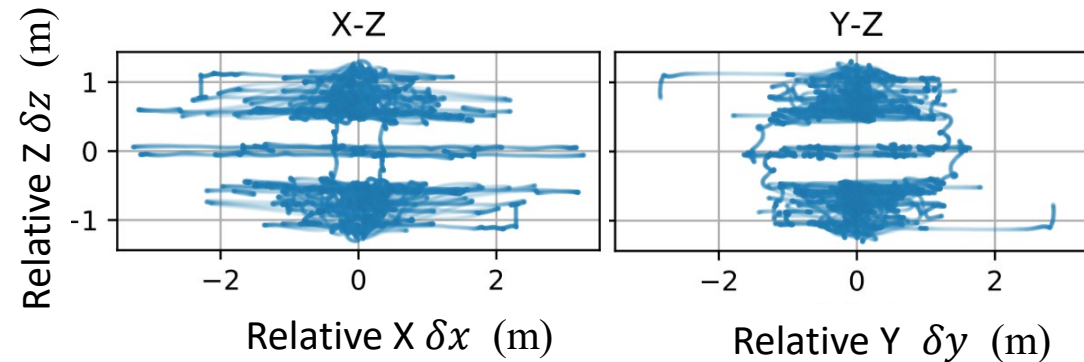
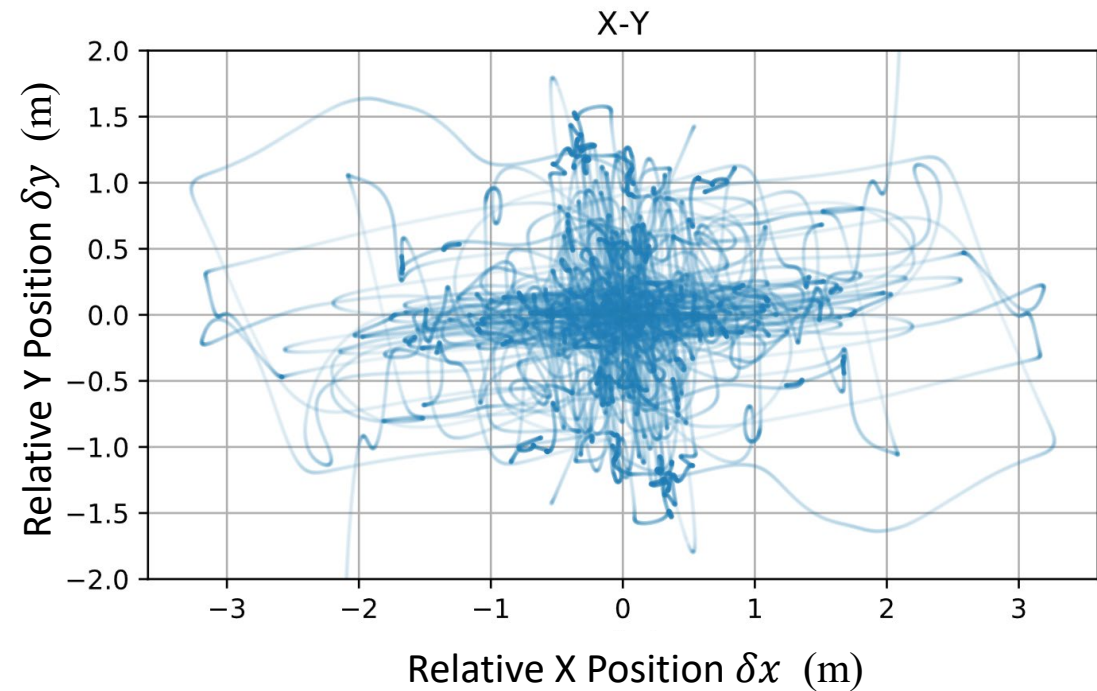
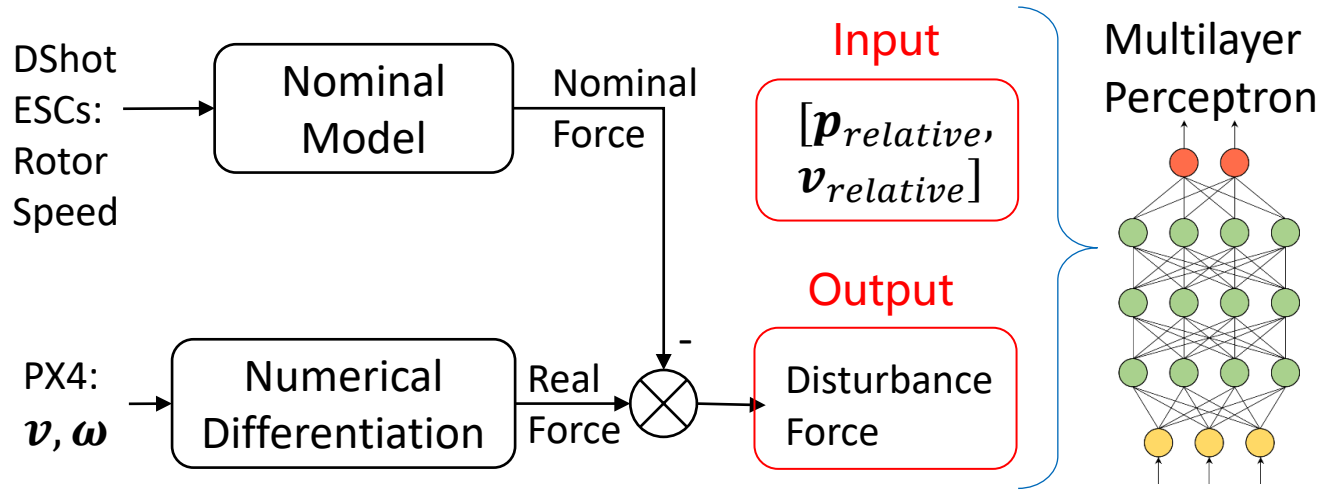
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Control Frequency:

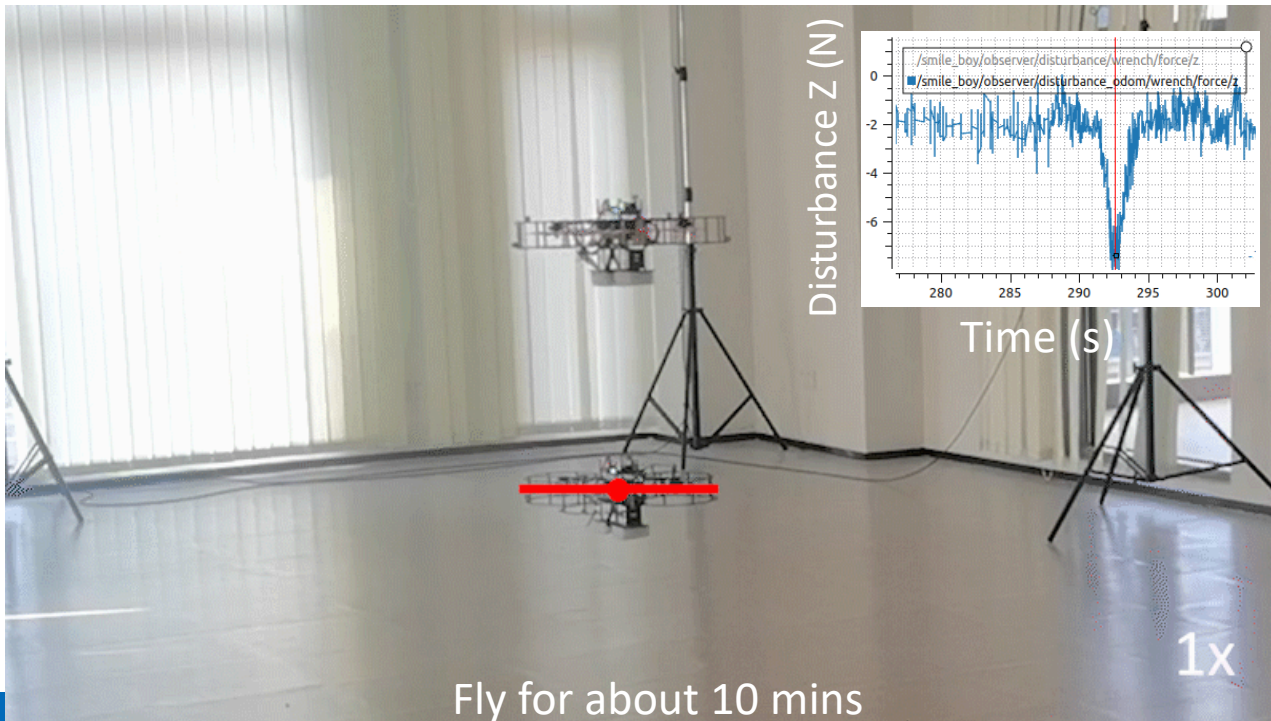
50~60Hz



# Collection of Disturbance Data



Total Data Number: 57,000 x 2



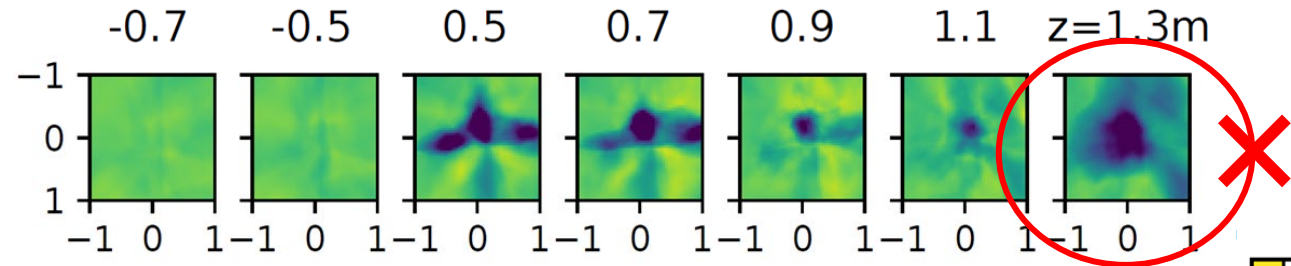
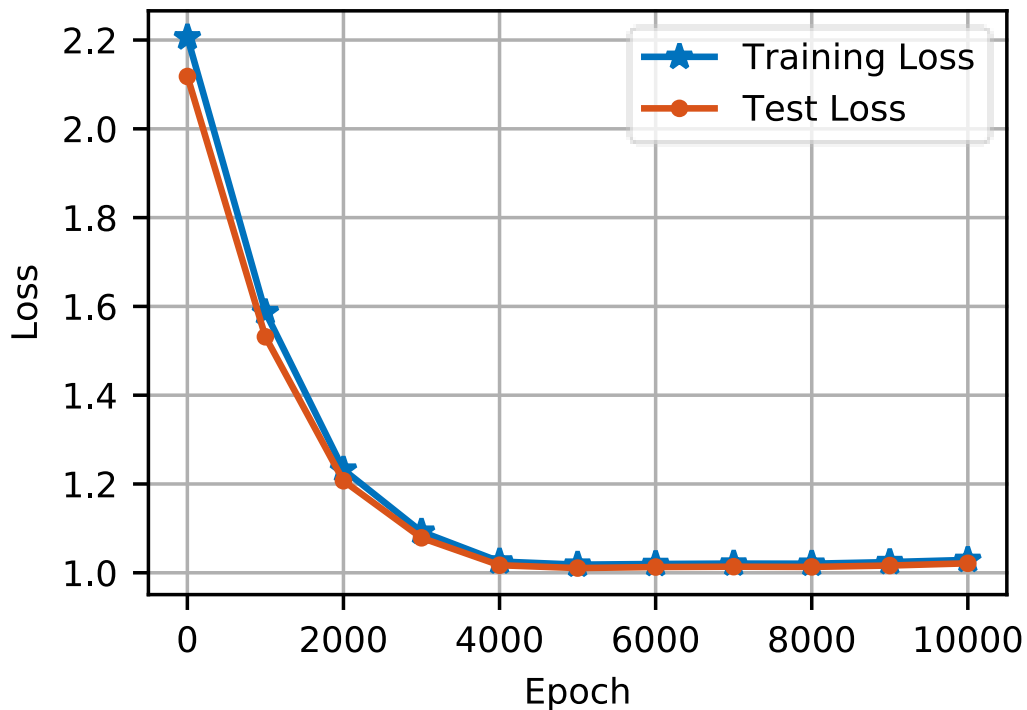
Fly for about 10 mins

# Training for Disturbance Network

Training Parameters

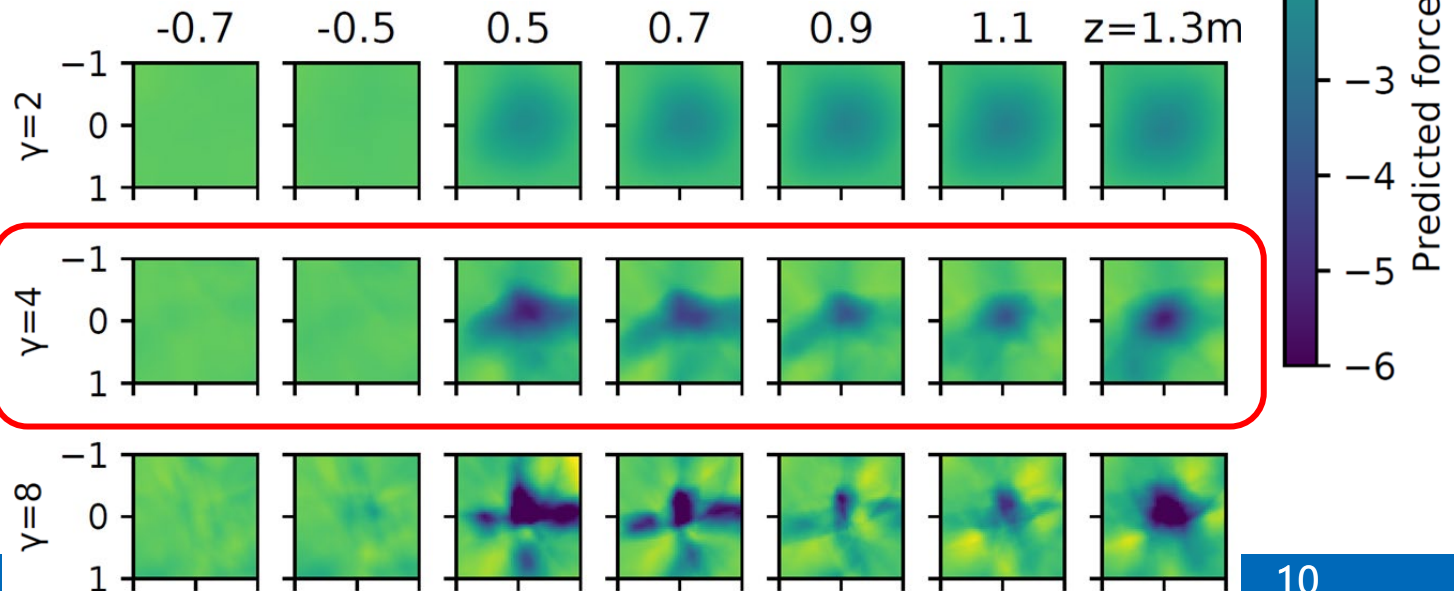
Layers	Initialization	Activation Function
6-128-64-128-3	normal	ReLU
Epoch	Learning Rate	Loss Function
20,000	1e-4	Mean Squared Error

Training Process when  $\gamma = 4$

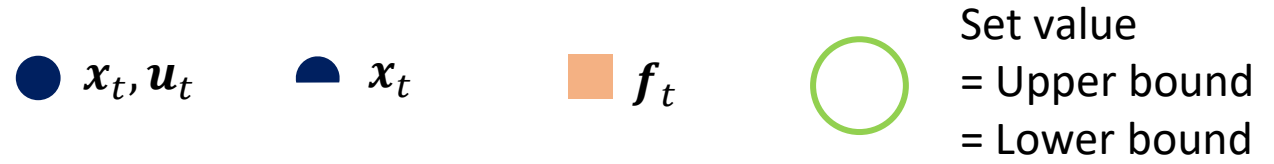


## Spectral Normalization

$$\begin{aligned} \|f(\mathbf{x}; \boldsymbol{\theta})\|_{\text{Lip}} &\leq \|W^{H+1} \mathbf{x} + \mathbf{b}^{H+1}\|_{\text{Lip}} \cdot \|\phi\|_{\text{Lip}} \cdot \|W^H \mathbf{x} + \mathbf{b}^H\|_{\text{Lip}} \cdots \|W^1 \mathbf{x} + \mathbf{b}^1\|_{\text{Lip}} \\ &= \|W^{H+1}\| \cdots \|W^1\| = \sigma_{\max}(W^{H+1}) \cdots \sigma_{\max}(W^1) = \prod_{l=1}^{H+1} \sigma_{\max}(W^l) \\ &\xrightarrow{\bar{W}^l := \gamma \cdot W^l / \sigma_{\max}(W^l)} \|f(\mathbf{x}; \bar{\boldsymbol{\theta}})\|_{\text{Lip}} \leq \gamma^{H+1} \end{aligned}$$



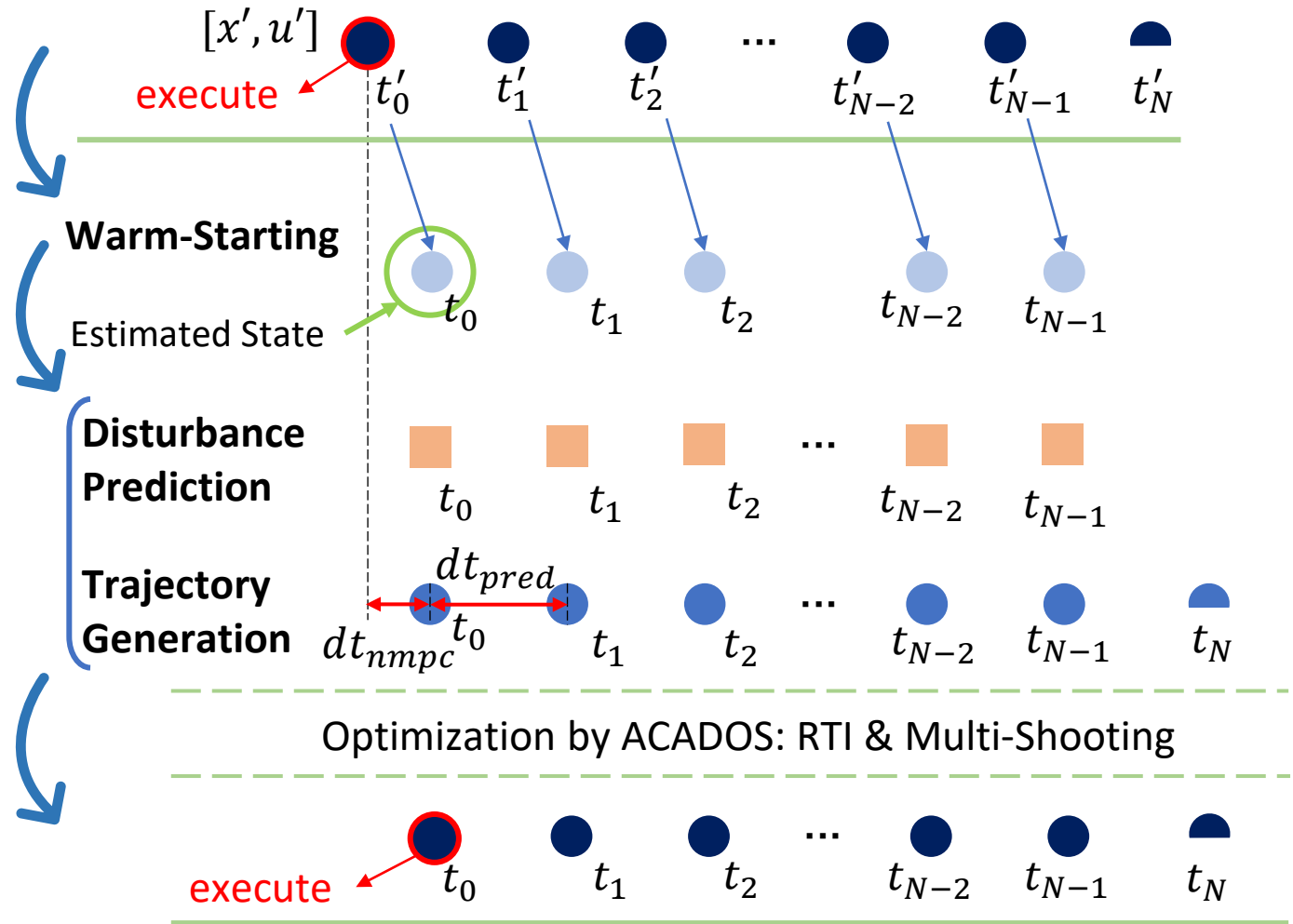
# Network Downwash Prediction NMPC (NDP-NMPC)



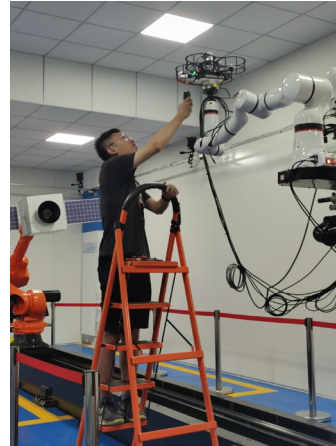
$$\begin{cases} {}^I \dot{\mathbf{p}} = {}^I \mathbf{v} \\ {}^I \dot{\mathbf{v}} = {}^I_B \mathbf{R}(\mathbf{q}) \cdot {}^B \mathbf{f}_u / m + {}^I \mathbf{g} \\ {}^B_I \dot{\mathbf{q}} = 1/2 \cdot {}^B_I \mathbf{q} \circ \mathcal{V}^* ({}^B \boldsymbol{\omega}) \end{cases}$$



$$\begin{cases} {}^I \dot{\mathbf{p}} = {}^I \mathbf{v} & \text{Disturbance Force} \\ {}^I \dot{\mathbf{v}} = ({}^I_B \mathbf{R}(\mathbf{q}) \cdot {}^B \mathbf{f}_u + {}^I \mathbf{f}_d) / m + {}^I \mathbf{g} \\ {}^B_I \dot{\mathbf{q}} = 1/2 \cdot {}^B_I \mathbf{q} \circ \mathcal{V}^* ({}^B \boldsymbol{\omega}) \end{cases}$$

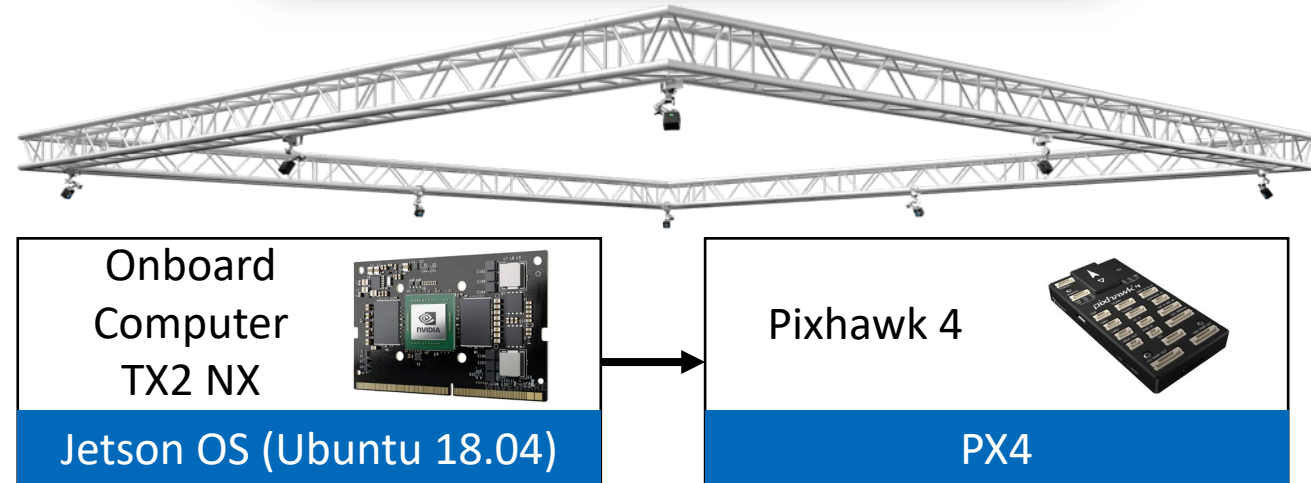


## Rotor Parameters

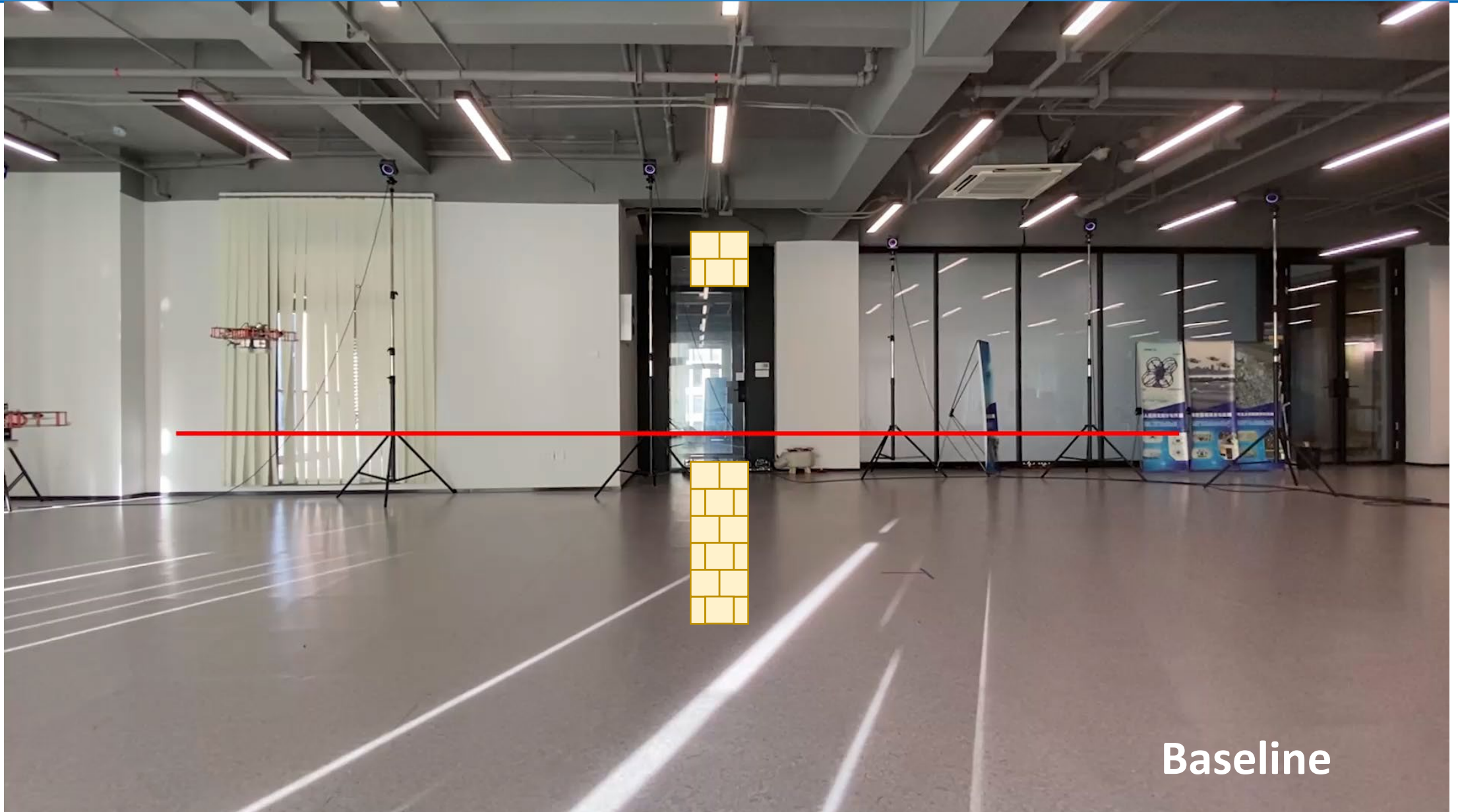


Parameter(s)	Value(s)	Unit
$L$	0.1372	m
$\alpha$	45	deg
$m$	1.5344	kg
$g$	9.81	m/s <sup>2</sup>
$I_{xx}$	0.0094	kg · m <sup>2</sup>
$I_{yy}$	0.0134	kg · m <sup>2</sup>
$I_{zz}$	0.0145	kg · m <sup>2</sup>
$k_q$	3.7611 E-4	N · m/kRPM <sup>2</sup>
$k_t$	2.8158 E-2	N/kRPM <sup>2</sup>
$[\Omega_{\min}, \Omega_{\max}]$	[2.6, 24.0]	kRPM
thrust/weight	4.3100	—
flight time	705	s

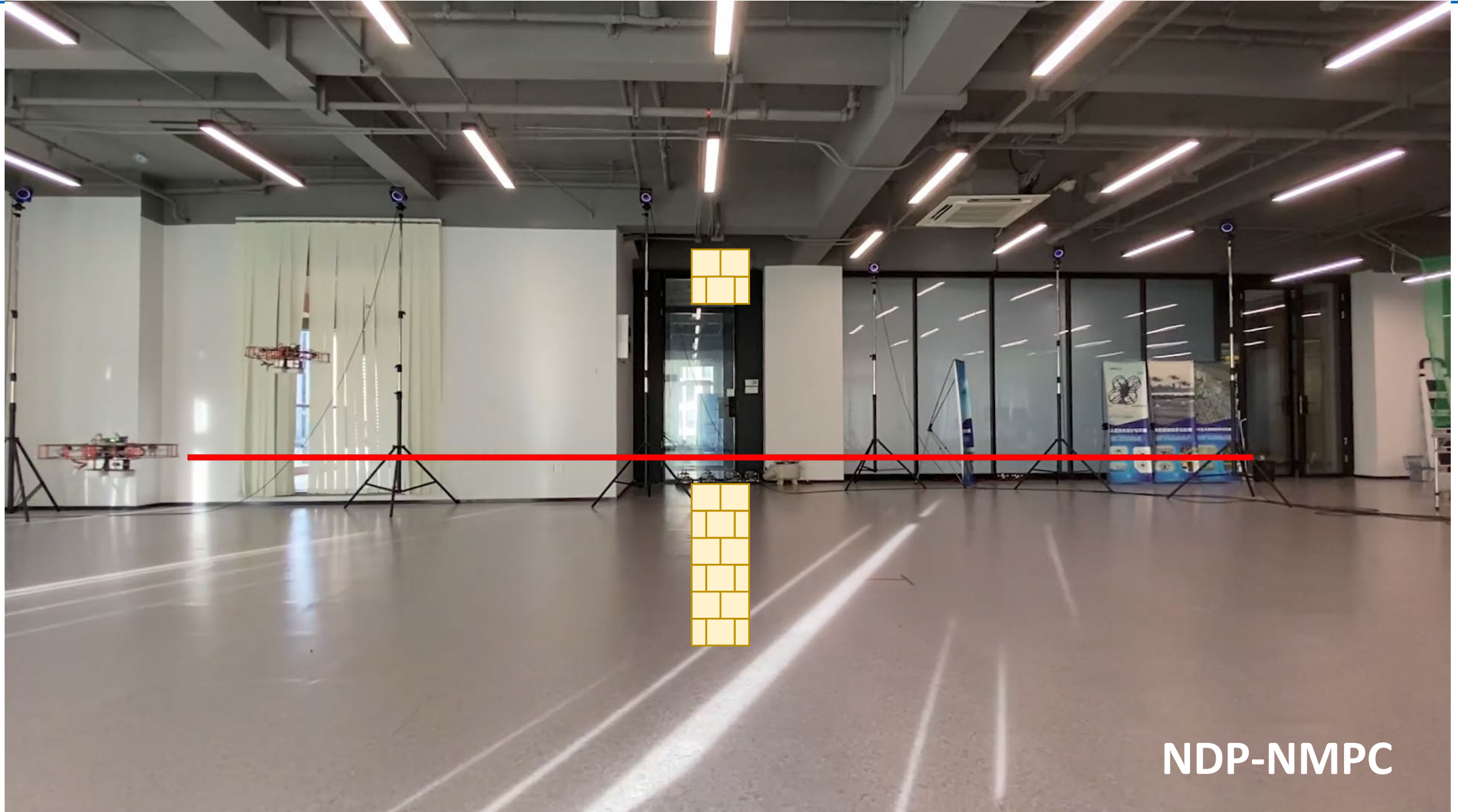
## Inertial Parameters (Bifilar Pendulum)



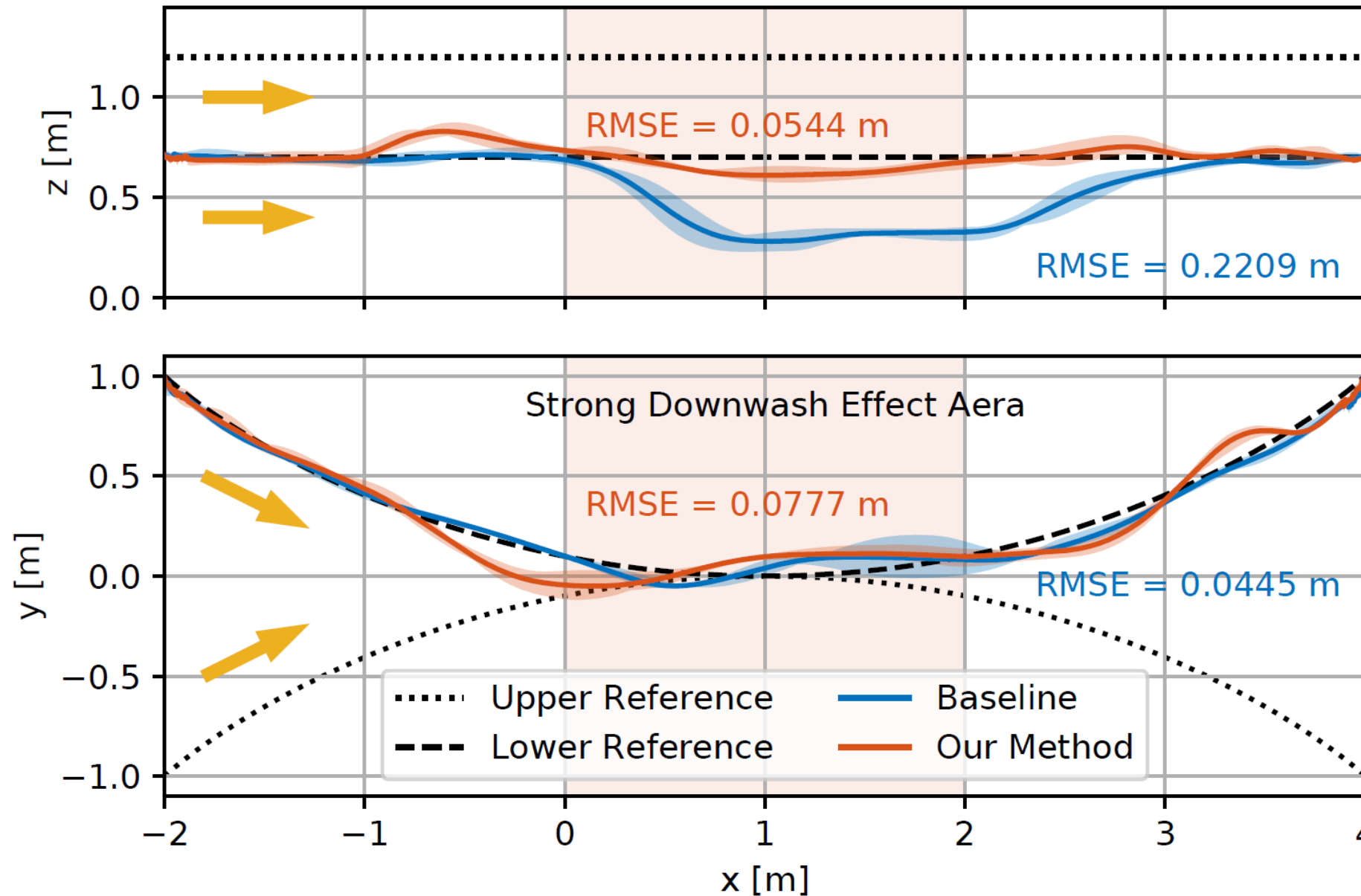
# Flight Experiment --- Baseline



# Flight Experiment --- NDP-NMPC



# Comparison of Results



↓ 75.37%

## Nonlinear MPC for Quadrotors in Close-Proximity Flight with Neural Network Downwash Prediction

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- Problem: downwash effect in two quadrotors
- Method: neural network + Nonlinear Model Predictive Control (NMPC)
- Achievement: two quadrotors, 0.5m close-proximity flight

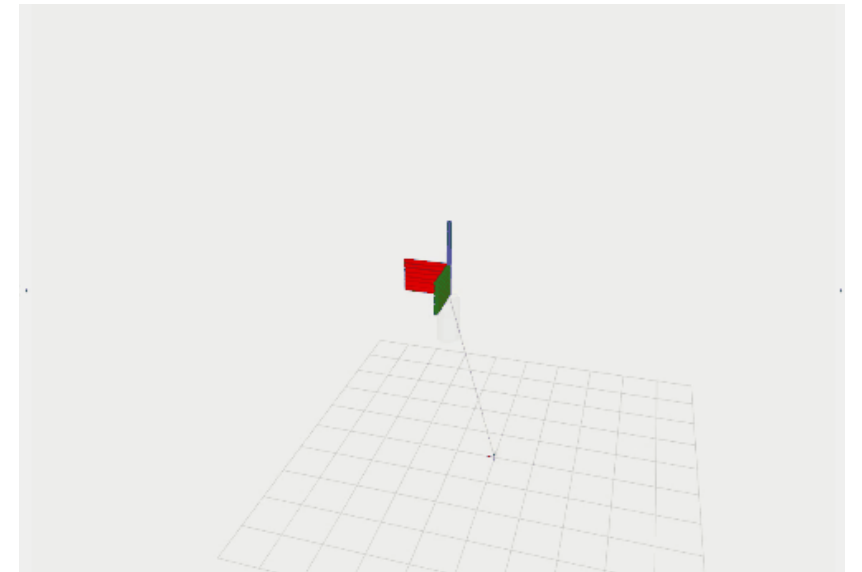
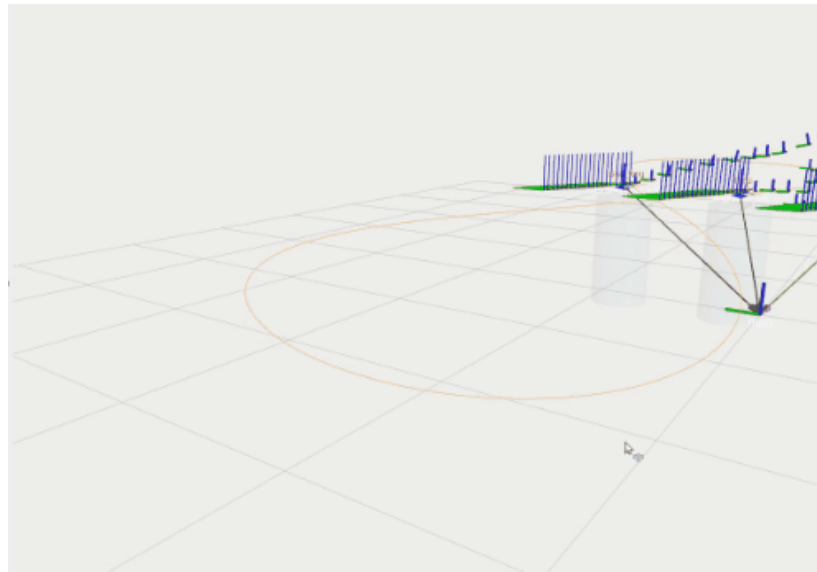
### Future Work

- Extend to more drones (Graph Neural Network, etc.....)
- Considering closed-loop disturbance estimation
- More extensive comparison with other control methods



# Thanks for listening!

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by Dr. Moju ZHAO, who created the DRAGON robot →



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