Servo Integrated Nonlinear Model Predictive Control For Overactuated Tiltable-Quadrotors

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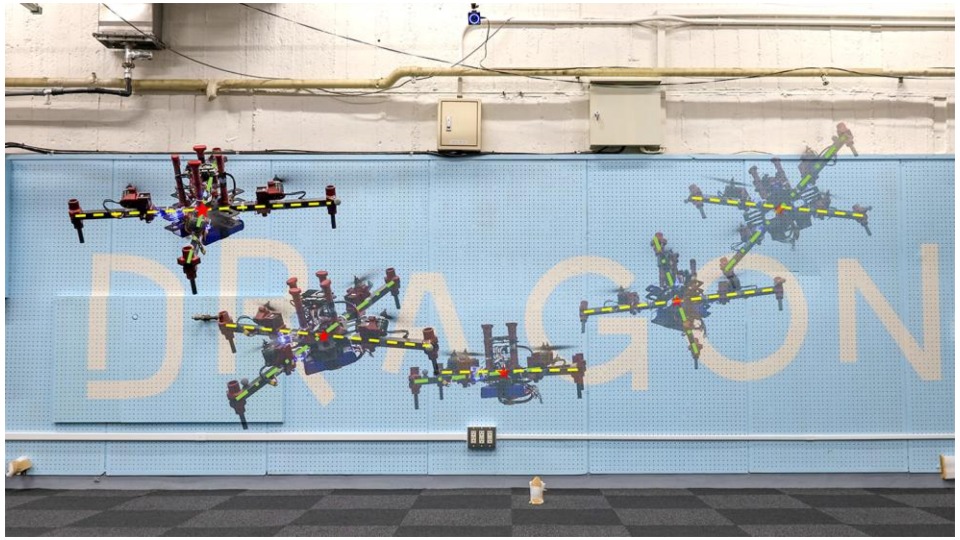

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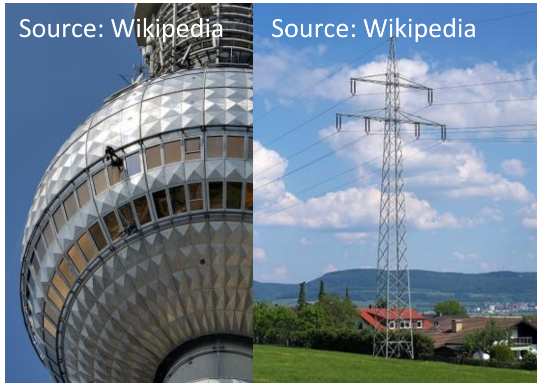
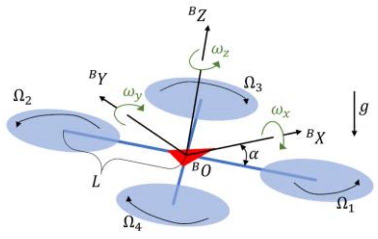
Introduction – Why Tiltable-Quadrotors?

Aerial Manipulation

Credit: DJI, MAVIC Air 2



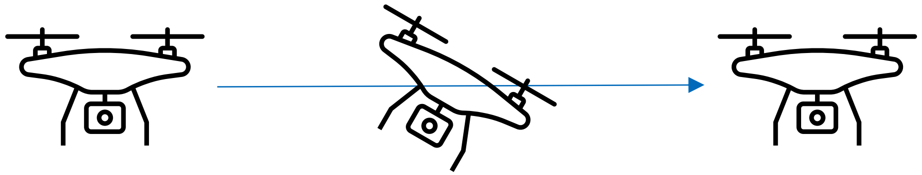
Full/Overactuation? Add **Servos**/Rotors



Quadrotor

Pros: Efficienc

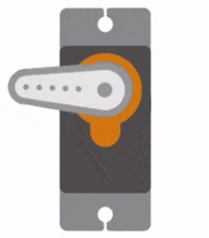
Quadrotors: Underactuated



Independent Position & Attitude ❌

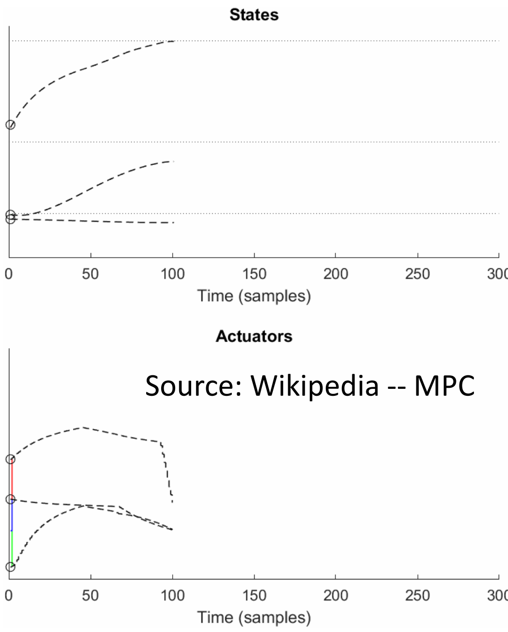
Reason: 4 (actuators) < 6 (DoF)

1. Nonlinearity ↑
2. Servo delay



Source: brainpad/due-servo

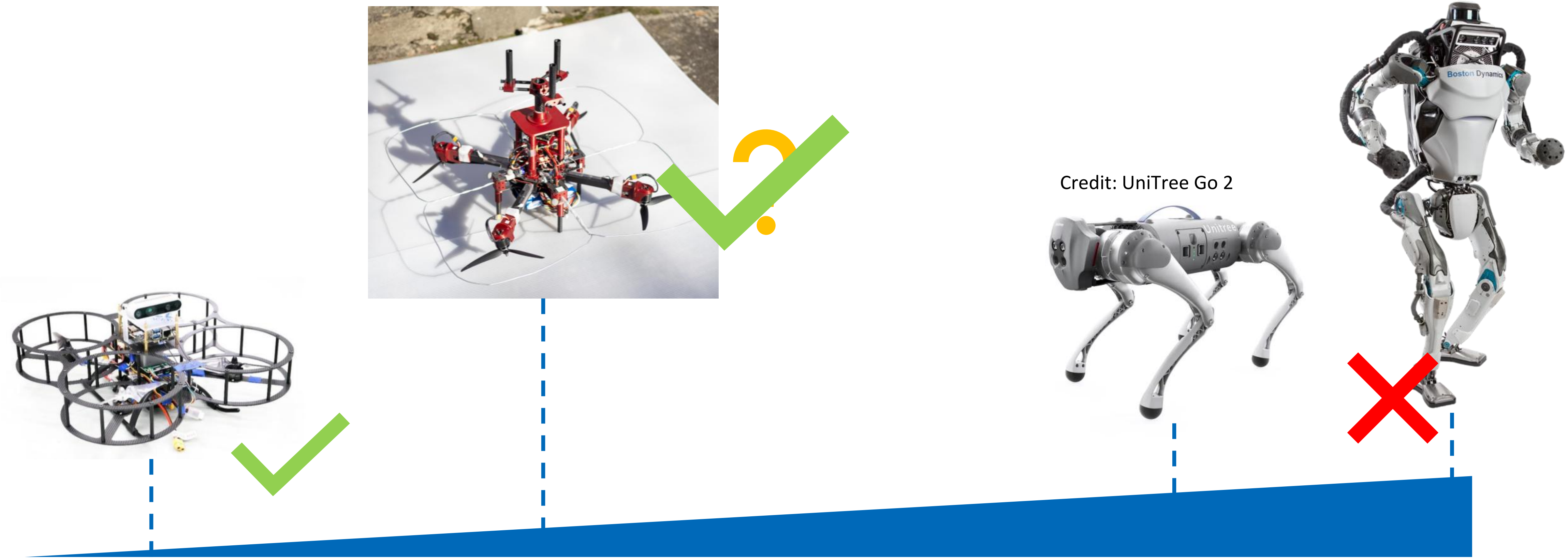
* Model Predictive Control (MPC)



Introduction --- Full-Model NMPC

The feasibility of NMPC depends on the model complexity.

Credit: Boston Dynamics Atlas

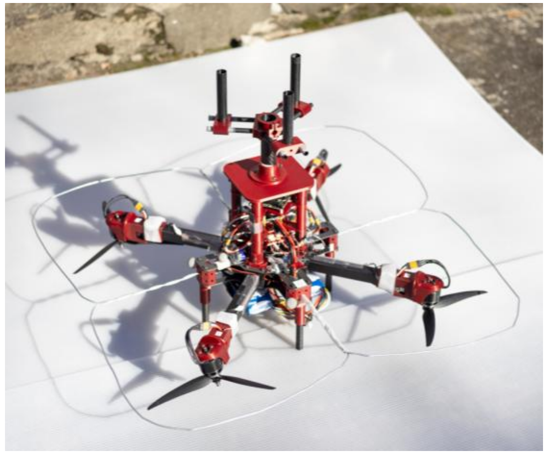


Sun et al., T-RO, 2022

Model nonlinearity and complexity

Robin Deits, RSS Workshop, 2024

Methodology



Tiltable-Quadrotor Model

$$\boldsymbol{x} = \left[{}^W\boldsymbol{p}, {}^W\boldsymbol{v}, {}^W_G\boldsymbol{q}, {}^G\boldsymbol{\omega}, \boldsymbol{\alpha}\right]^T \quad \boldsymbol{u} = \left[\boldsymbol{f}_c, \boldsymbol{\alpha}_c\right]^T$$

Rotor Model

$$f_i = k_t\,\Omega_i^2, \quad \tau_i = k_q\,\Omega_i^2,$$

$${}^{R_i}\boldsymbol{f}_i = [0,0,f_i]^T, \quad f_i \in [f_{i,\min}, f_{i,\max}],$$

$${}^{R_i}\boldsymbol{\tau}_i = \left[0, 0, -d_i\, f_i\,\frac{k_q}{k_t}\right]^T.$$

Servo Model

$$\dot{\boldsymbol{\alpha}} = \frac{1}{t_{\text{servo}}}\left(\boldsymbol{\alpha}_c - \boldsymbol{\alpha}\right),$$

Resultant Wrench

$${}^G\boldsymbol{f}_u = \sum_{i=1}^{N_p} {}^G_{E_i}\boldsymbol{R}\,{}^{E_i}_{R_i}\boldsymbol{R}\,{}^{R_i}\boldsymbol{f}_i,$$

$${}^G\boldsymbol{\tau}_u = \sum_{i=1}^{N_p}\left({}^G_{E_i}\boldsymbol{R}\,{}^{E_i}_{R_i}\boldsymbol{R}\,{}^{R_i}\boldsymbol{\tau}_i + {}^G\boldsymbol{p}_{r,i} \times {}^G_{E_i}\boldsymbol{R}\,{}^{E_i}_{R_i}\boldsymbol{R}\,{}^{R_i}\boldsymbol{f}_i\right),$$

Rigid-Body Model

$${}^W\dot{\boldsymbol{p}} = {}^W\boldsymbol{v},$$

$${}^W\dot{\boldsymbol{v}} = \left({}^W_G\boldsymbol{R}(\boldsymbol{q})\,{}^G\boldsymbol{f}_u + {}^W\boldsymbol{f}_d\right)/m + {}^W\boldsymbol{g},$$

$${}^W_G\dot{\boldsymbol{q}} = \frac{1}{2}\,{}^W_G\boldsymbol{q} \circ \mathcal{H}\left({}^G\boldsymbol{\omega}\right),$$

$${}^G\dot{\boldsymbol{\omega}} = \boldsymbol{I}^{-1}\left(-{}^G\boldsymbol{\omega} \times \left(\boldsymbol{I}\,{}^G\boldsymbol{\omega}\right) + {}^G\boldsymbol{\tau}_u + {}^G\boldsymbol{\tau}_d\right),$$

Nonlinear Optimization Problem

$$\underset{\boldsymbol{x}_k, \boldsymbol{u}_k}{\text{minimize}} \sum_{k=0}^{N-1}\left(\overline{\boldsymbol{x}}_k^T \boldsymbol{Q} \overline{\boldsymbol{x}}_k + \overline{\boldsymbol{u}}_k^T \boldsymbol{R} \overline{\boldsymbol{u}}_k\right) + \overline{\boldsymbol{x}}_N^T \boldsymbol{Q}_N \overline{\boldsymbol{x}}_N,$$

→ Nonlinear Least Square Cost; SQP

subject to

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right), \quad k = 0 : N-1,$$

→ Dynamics Constraint

$$\boldsymbol{x}_0 = \hat{\boldsymbol{x}},$$

→ Initial Value Constraint

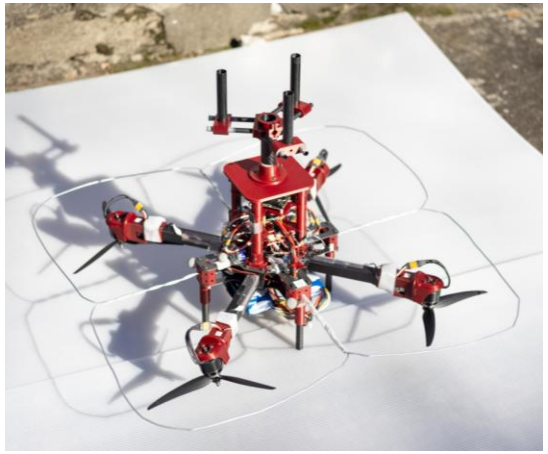
$$\|v_{x,y,z}\| \leq v_{\text{limit}}, \quad \|\omega_{x,y,z}\| \leq \omega_{\text{limit}},$$

→ State Constraint

$$\boldsymbol{u}_{\min} \leq \boldsymbol{u}_k \leq \boldsymbol{u}_{\max},$$

→ Input Constraint

Methodology



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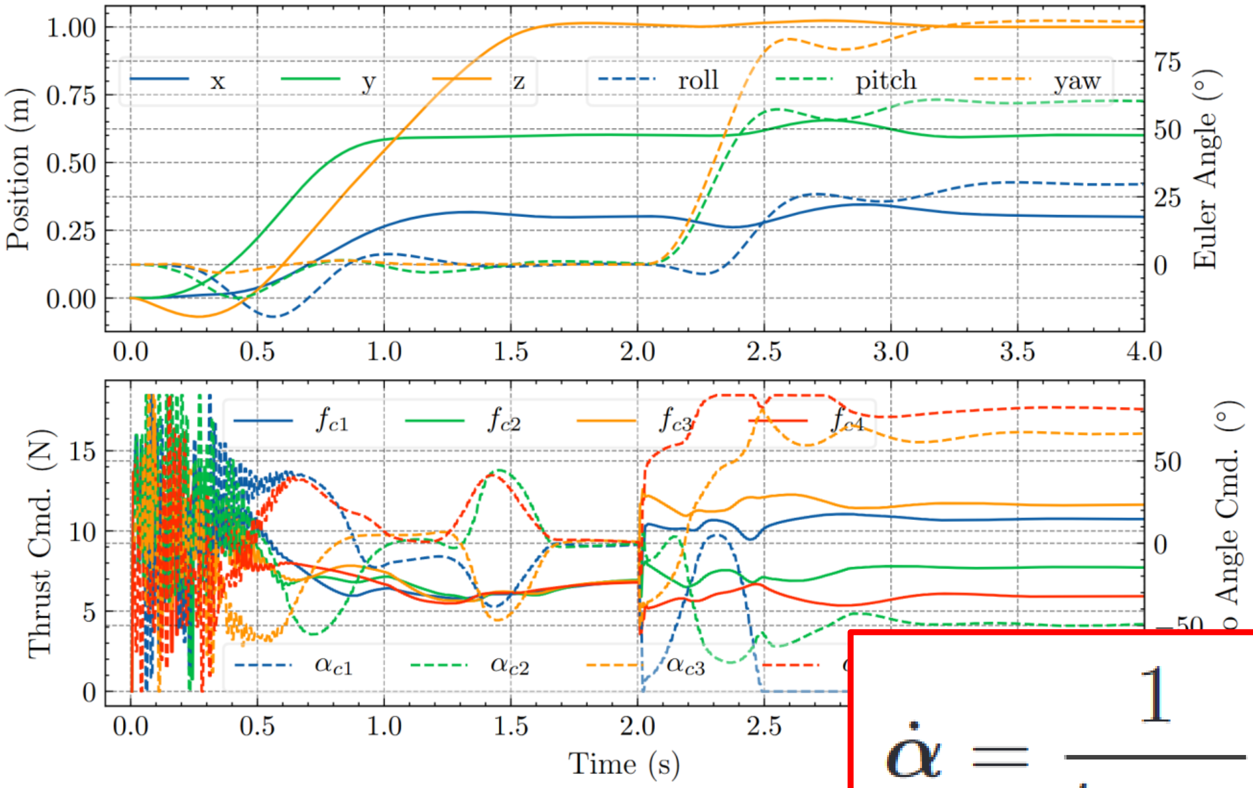
Does it really matter?

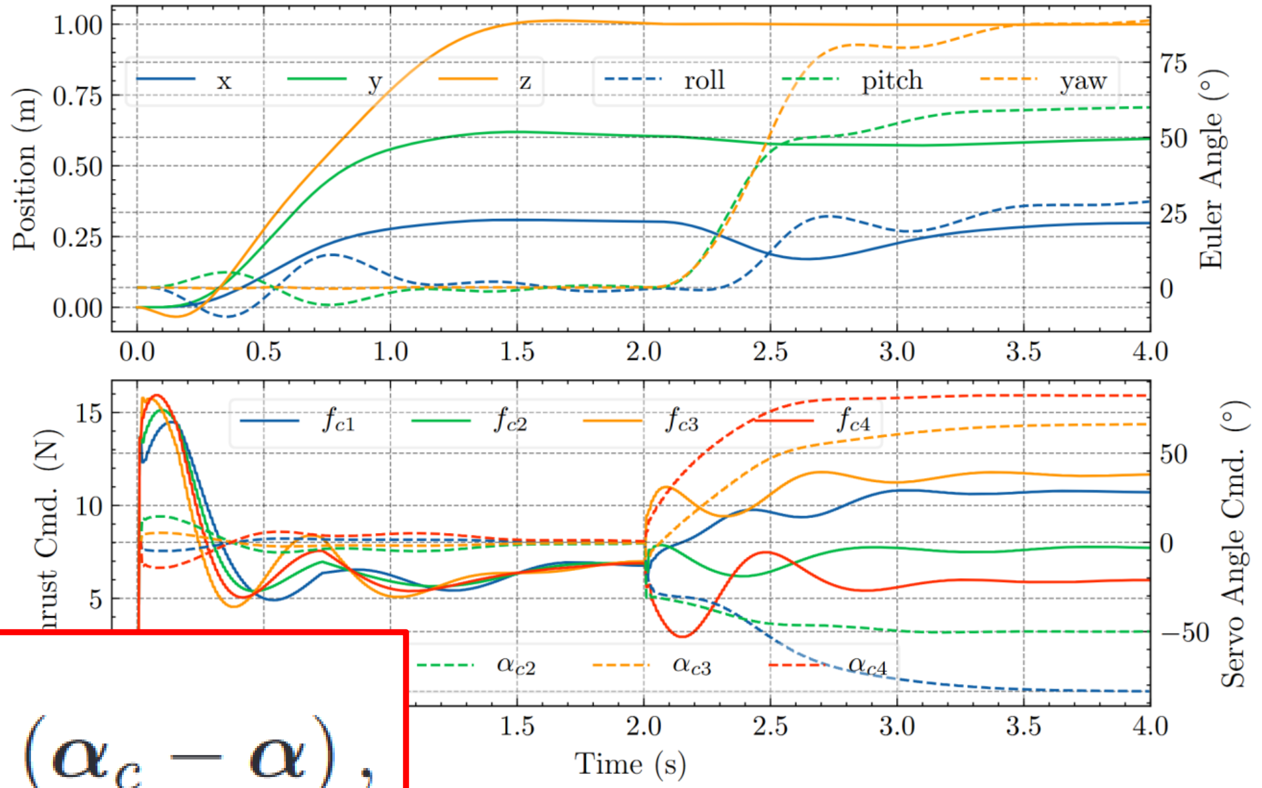
Simulation – The Effect of Servo Dynamics

- Ideal Simulation: No disturbance, no model error
- Rigid-Body Dynamics + Servo Dynamics + Rotor Dynamics
- All parameters = real robot

t=0s, Position → $[0.3, 0.6, 1.0]^T$ m
t=2s, Attitude → $[30°, 60°, 90°]^T$



NMPC ❌ servo model



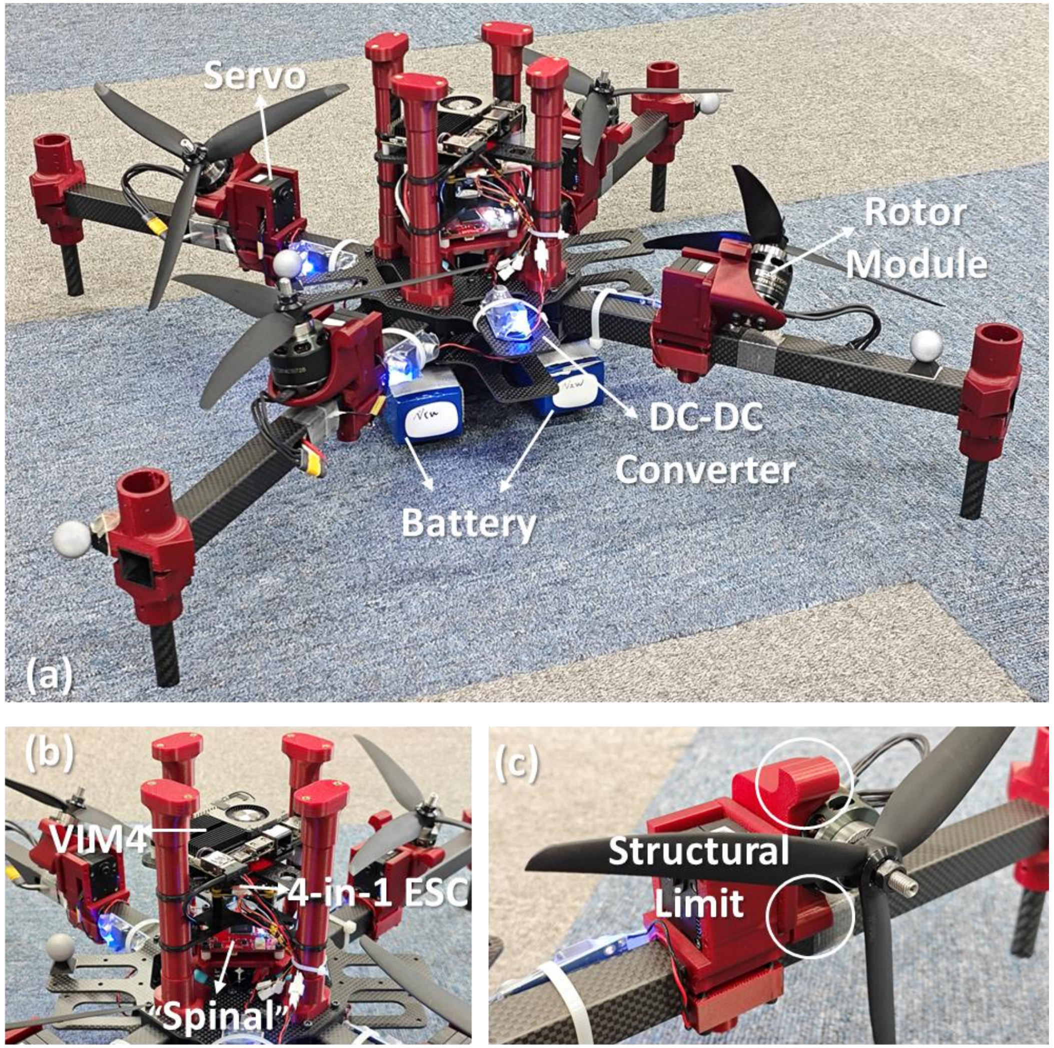
NMPC ✔ servo model

$$\dot{\alpha} = \frac{1}{t_{\mathrm{servo}}} \left(\alpha_c - \alpha \right),$$

Reason: servo model ≈ continuity constraint

Robot

Beetle-Art

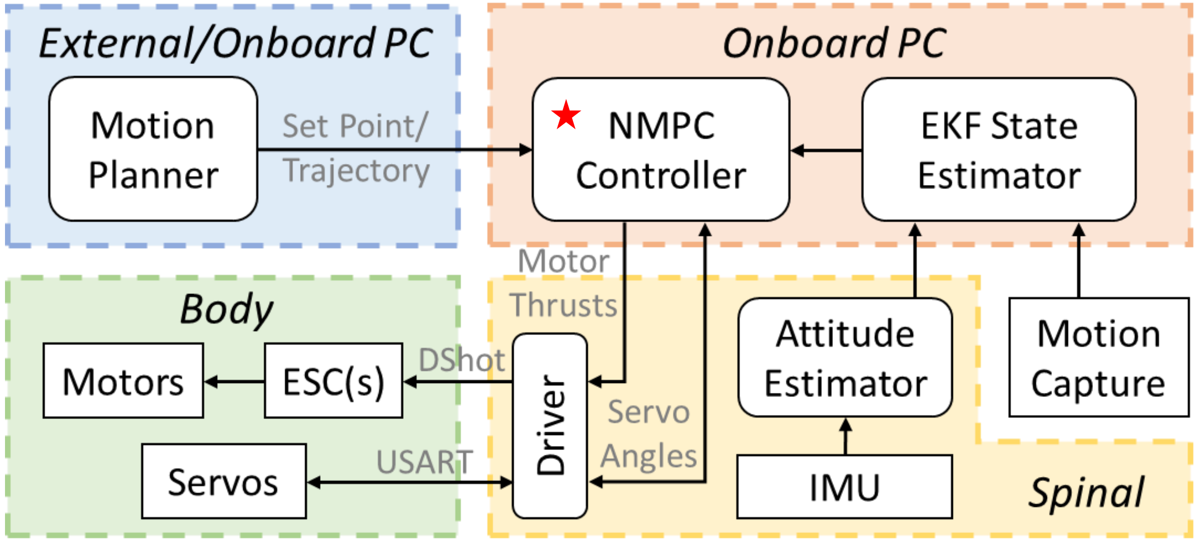


Basic Parameters

Mass: 2.773 kg
Wheelbase: 0.4 m
T (servo): 0.0859 s (*Identified*)
Freq. (control): **100 Hz**

Param.	Value	Param.	Value	Param.	Value
Wheelbase	0.4 m	m	2.773 kg	α_{limit}	$\pm\pi/2$
I_{xx}	0.0417	I_{yy}	0.0395	I_{zz}	0.0707 kg m^2
N_p	4	k_q/k_t	0.0153	t_{servo}	0.0859 s
N	20	t_{integ}	0.1 s	t_s	0.01 s
$Q_{p,\text{xy}}$	300	$Q_{p,\text{z}}$	400	$Q_{\text{v,xy}}$	10
$Q_{v,\text{z}}$	10	$Q_{q,\text{xy}}$	300	$Q_{\text{q,z}}$	600
$Q_{\omega,\text{xy}}$	5	$Q_{\omega,\text{z}}$	5	Q_α	2
R_f	2	R_α	250		
v_{limit}	±1 m/s	ω_{limit}	±6 rad/s	$\alpha_{i,\text{limit}}$	$\pm\pi/2$
$f_{i,\text{min}}$	0 N	$f_{i,\text{max}}$	30 N	$\alpha_{ci,\text{limit}}$	$\pm\pi/2$
$k_{I,z}$	5	$f_{d,\text{limit}}$	5 N		

System Overview

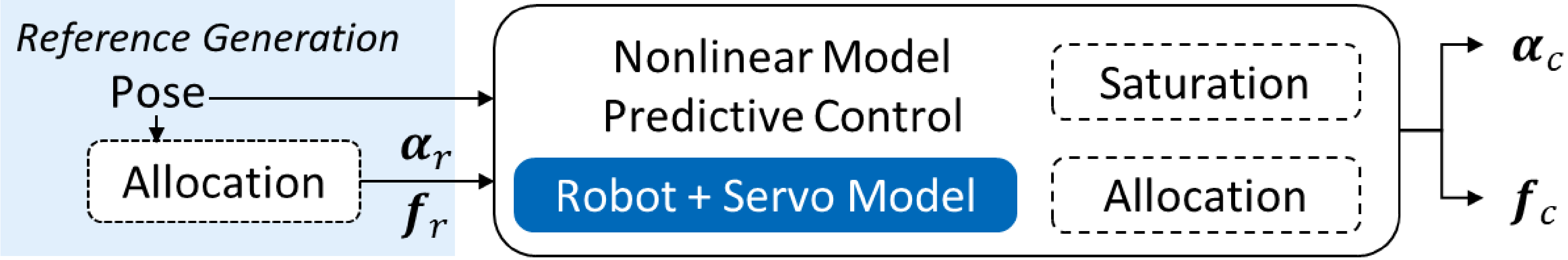


Omnidirectional Traj. Tracking

Conclusion

Key Takeaways

- Full-Model NMPC → Tiltable-quadrotors ✅
- Servo matters; Impact: Servo > Rotor
- First time, Real experiment, 100Hz



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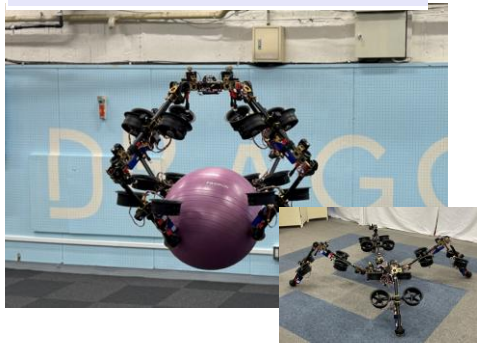
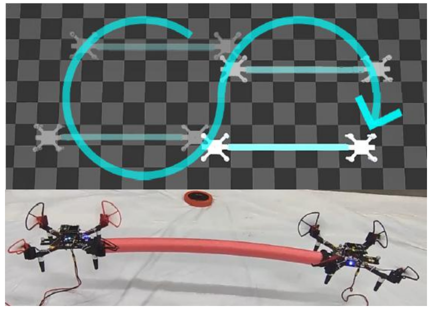
ATLANTA 2025

Other papers from our lab

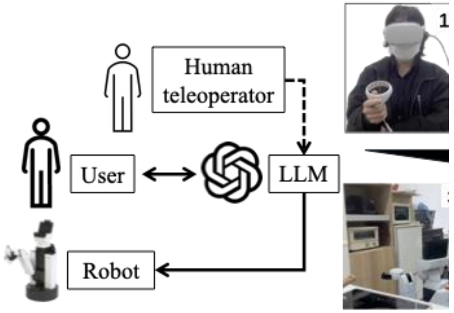
WeAT5
Aerial Manipulation 1
08:45-08:50, Paper WeAT5.4



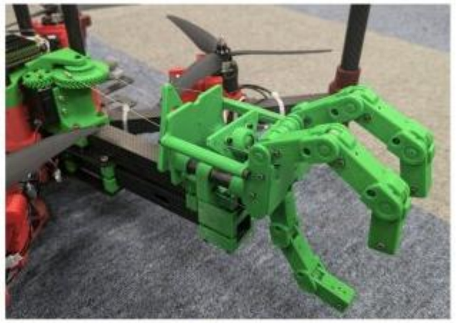
WeBT5
Aerial Robots: Mechanics and Control 1
10:20-10:25, Paper WeBT5.6

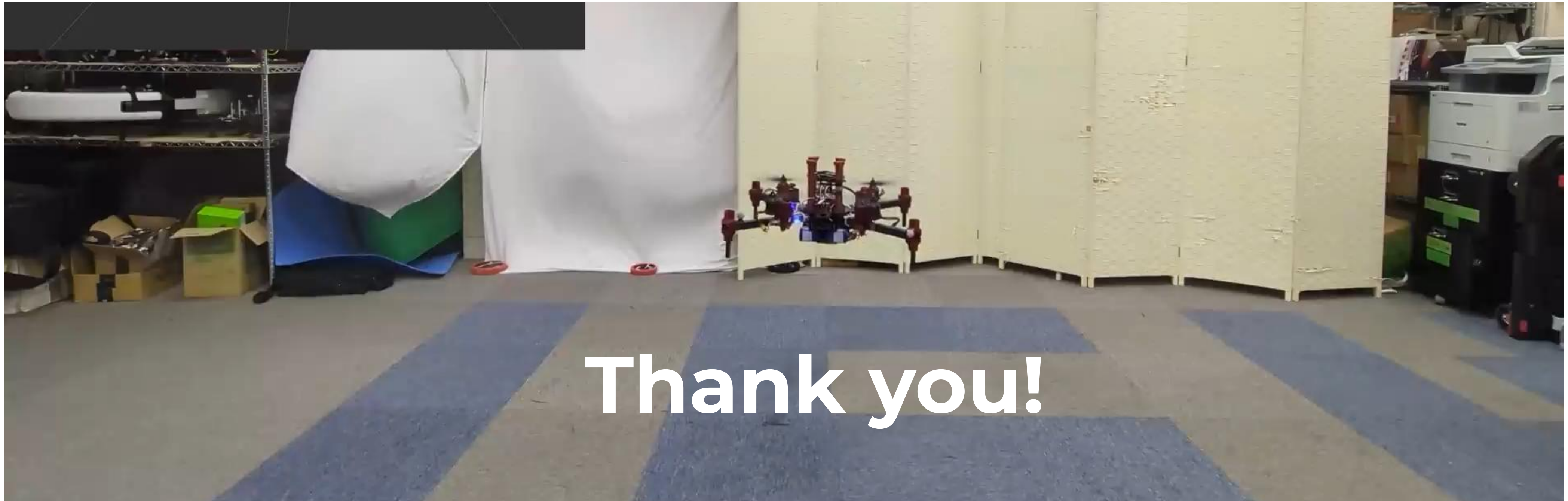


WeDT11
Foundation Models for Manipulation
15:15-15:20, Paper WeDT11.1



ThDT14
End-Effectors
15:25-15:30, Paper ThDT14.3





Thank you!